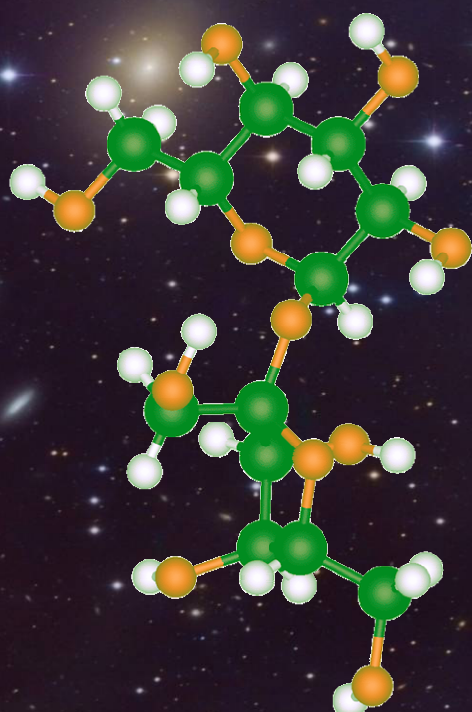
The Hierarchy Problem



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Why is the Universe so big ?

Size of Universe:	$5\times10^{26}\ m$
Size of stars and planets:	$10^{7}-10^{10}\ m$
Size of humans:	$1\ m$
Size of Atoms:	$10^{-10}\ m$
Planck size:	$10^{-35}\ m$

Super String Theory

$$h / 2\pi = \hbar = 1.0546 \times 10^{-34} \ \text{kg m}^2 \ \text{sec}^{-1}$$

$$G_N = 6.672 \times 10^{-11} \text{m}^3 \ \text{kg}^{-1} \ \text{sec}^{-2}$$

$$c = 2.99792458 \times 10^8 \ \text{m / sec}$$

Planck Units

$$L_{\text{Planck}} = \sqrt{\frac{\hbar \ G_N}{c^3}} = 1.616 \times 10^{-33} \text{ cm}$$

$$M_{\text{Planck}} = \sqrt{\frac{\hbar c}{G_N}} = 21.8 \ \mu \text{ g}$$

$$T_{\text{Planck}} = \sqrt{\frac{\hbar \ G_N}{c^5}} = 5.39 \times 10^{-44} \ \text{sec}$$

Many other constants make *dimensionless combinations*:

Fine structure constant:

$$\alpha = e^2 / 4\pi\hbar c = 1/137.036$$

$$\frac{\text{proton mass}}{\text{Planck units}} : \quad \kappa = m_P \sqrt{G / \hbar c} = 7.685 \times 10^{-20}$$

$$\frac{\text{proton mass}}{\text{electron mass}} : \quad \frac{m_P}{m_e} = 1836.1527$$

$$\frac{\text{Cosmological Constant}}{\text{Planck units}} : \quad \frac{\Lambda \hbar G}{c^5} = 3 \times 10^{-122}$$

Size atomic nucleus: $$10\frac{\hbar}{m_P\, c} \approx 2\times10^{-15}\, m$$

Size of atoms : $$\frac{\hbar}{\alpha\, m_e\, c} \approx 0.5\times10^{-10}\, m$$

Density of rock: $$\approx m_P\left(\frac{\alpha\, m_e c}{\hbar}\right)^3 \approx 11\, g\,/\,cm^3$$

Mass of planet (chemical):

$$\sqrt{\frac{\alpha^3 c^3 \hbar^3}{m_P^4\, G^3}} \approx 384\, M_{\text{Earth}}$$

Mass of planet (chemical): $\sqrt{\dfrac{\alpha^3 c^3 \hbar^3}{m_P^4 G^3}} \approx 384\ M_{\text{Earth}}$

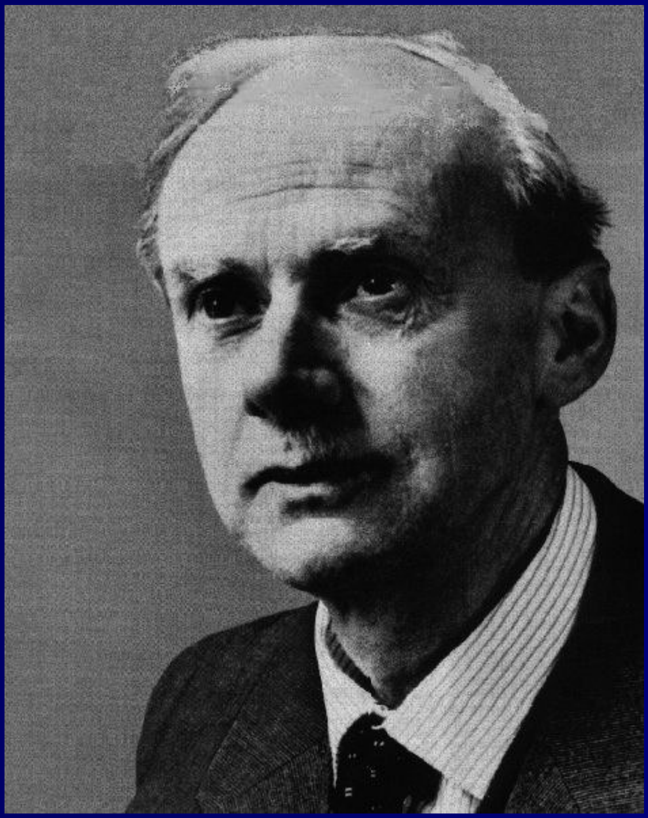
$\dfrac{\text{mass of star (nuclear)}}{\text{mass of planet (chemical)}}$:

$$\left(\frac{m_P}{m_e\ \alpha^2}\right)^{3/4} \approx 450\,000$$

$\dfrac{\text{“ Size of the universe”}}{\text{Planck length}}$: $\sqrt{\dfrac{c^5}{\Lambda\ \hbar G}} = 10^{61}$

It appears that the Universe is so big because the constants of Nature take wildly varying values.
But why do these values vary so much?

HIERARCHY PROBLEM



P.A.M. Dirac:
Perhaps the Universe is so big because it is so old,

so one may suspect that constants of nature today vary wildly because they all slowly change with time.

$$t \approx 10^{61} t_0 \quad ; \qquad \frac{\Lambda \hbar G}{c^5} \approx \frac{t_0^2}{t^2}$$

$$\frac{1}{\alpha} \approx \log\left(\frac{t}{t_0}\right) \approx 140$$

$$\frac{m_P}{M_{\text{Planck}}} \approx \left(\frac{t_0}{t}\right)^{1/3} \approx 10^{-20}$$

$$\frac{m_P}{m_e} \approx \left(\log(t / t_0)\right)^{3/2} \approx 1650$$

Experimental checks

Does α vary with space and time?

$$1/\alpha = 137.03599968(9)$$

Accuracy about .37 ppb.

According to Dirac, this value would increase as $\delta t / t$, which would not be noticeable within 100 years.

But *changes* can be detected much more accurately!

Hyperfine splittings of spectral lines get relativistic corrections that are different for different species of atoms.

These differences depend on α .

Compare spectral lines (hyperfine splittings) of Hg^{2+} and H .

It was found that

$$\left|\frac{\dot{\alpha}}{\alpha}\right| \leq 3.7\times10^{-14} \text{ / year}$$

while the universe is only 1.37×10^{10} years old.

This already contradicts Dirac.

And one can do even better :

The Oklo mine in Gabon

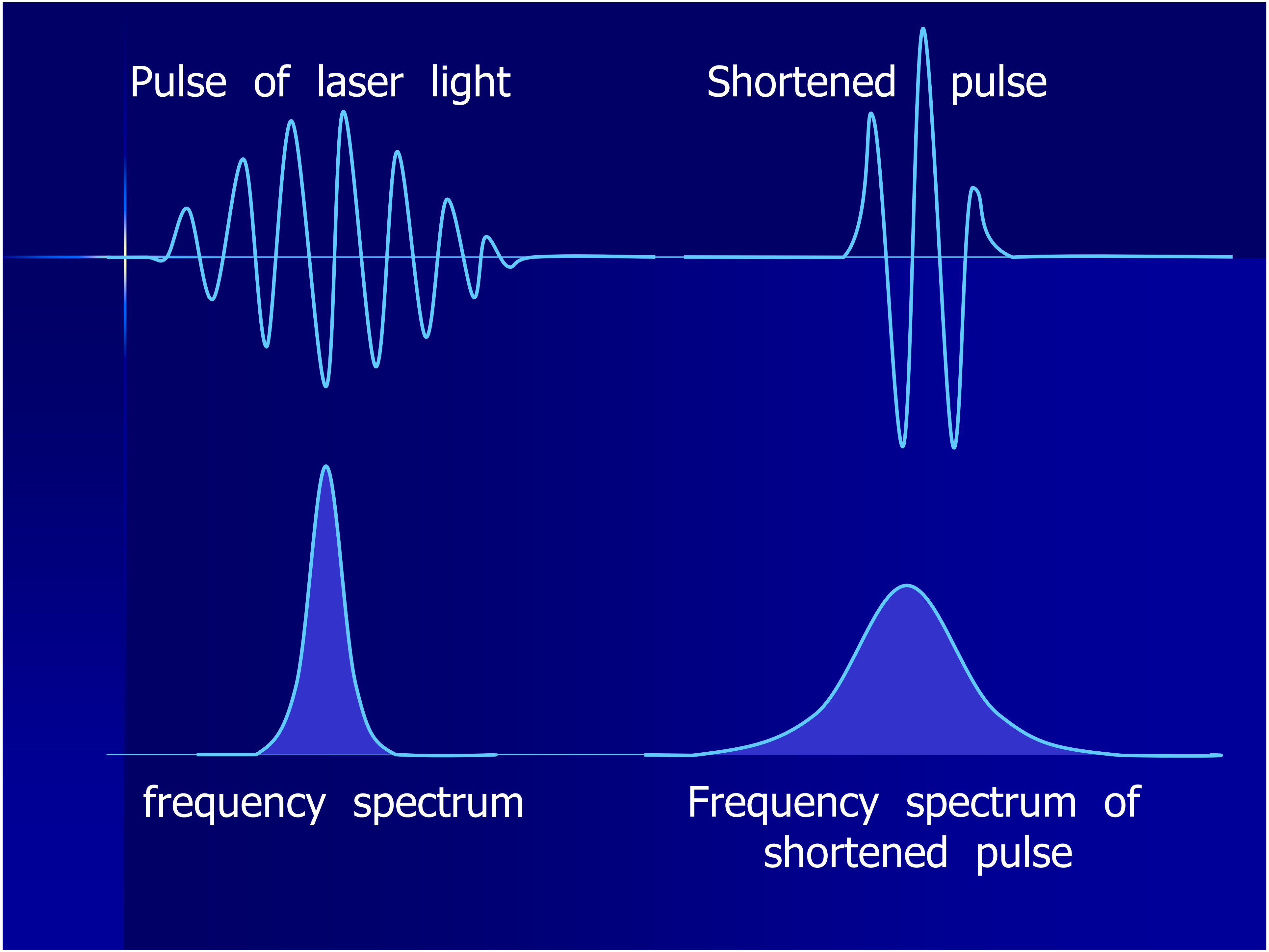
Isotope ratio $^{149}Sm/^{147}Sm$ deviates from the naturally occurring value: this has been a nuclear reactor, 2 billion years ago.

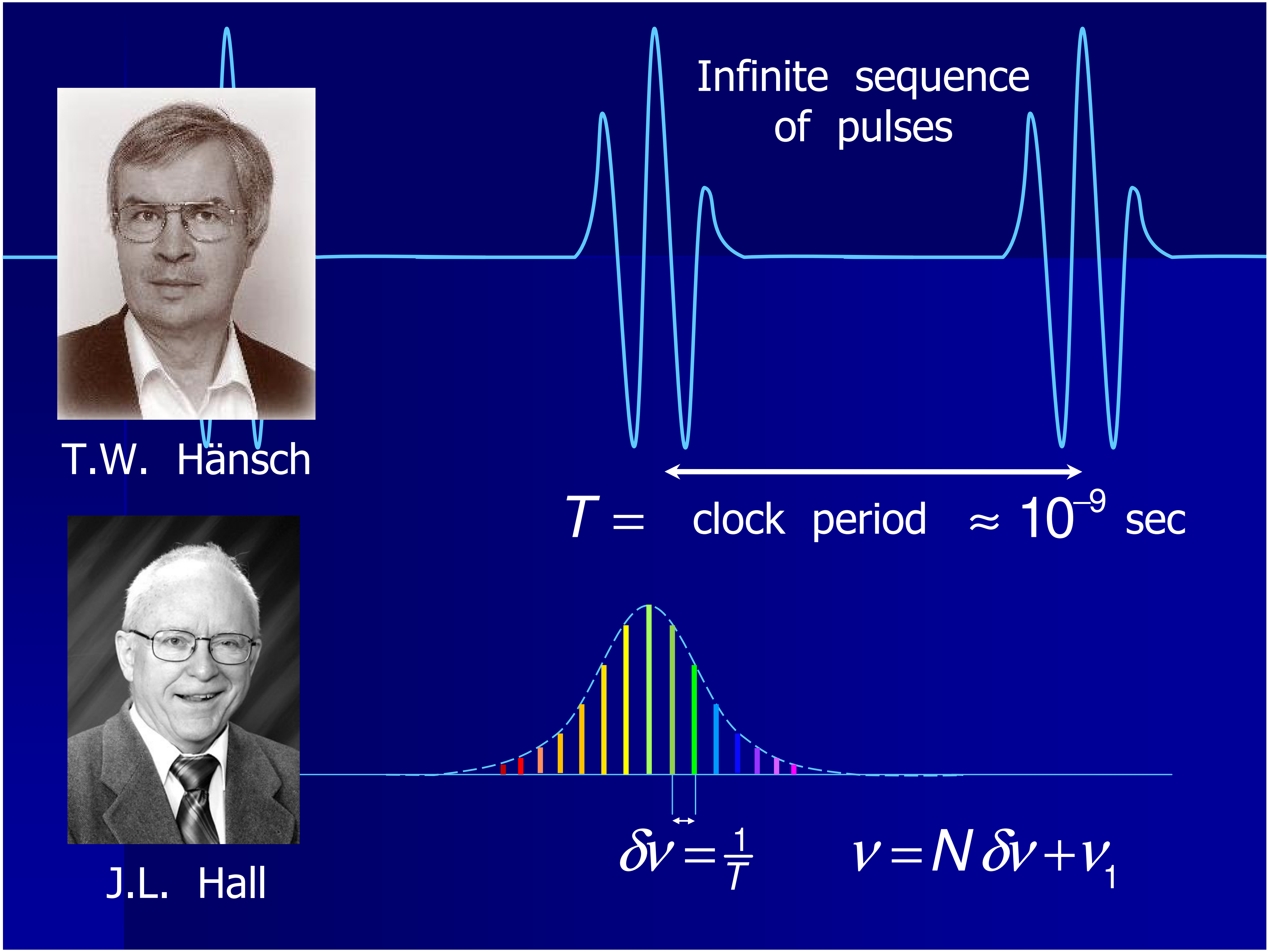
$$\left|\frac{\dot{\alpha}}{\alpha}\right| \leq 10^{-15} \text{ / year}$$

One can do better still:

Use the *frequency comb* :

1. Make short laser pulses.


Pulse of laser light
Shortened pulse
frequency spectrum
Frequency spectrum of shortened pulse


Infinite sequence of pulses
T.W. Hänsch
$T =$ clock period $\approx 10^{-9}$ sec
J.L. Hall
$\delta\nu = \frac{1}{T}$
$\nu = N\,\delta\nu + \nu_1$

Comparing spectral lines of Al^+ and Hg^+, it was found that

$$\frac{\dot{\alpha}}{\alpha} \approx -1.6 \pm 2.3 \times 10^{-17} \text{ / year}$$

Space dependent variations of α :

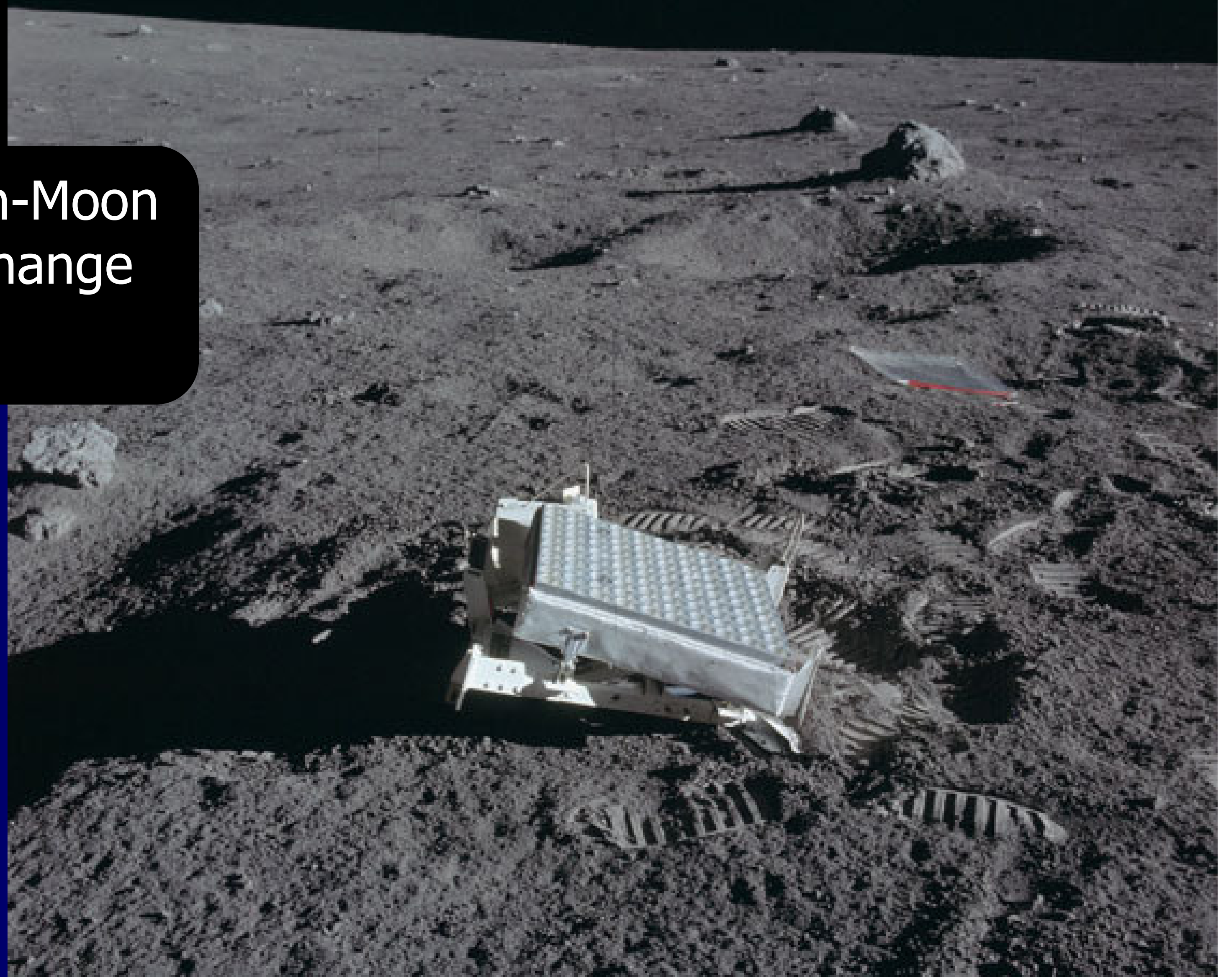
Comparison of spectral lines very distant galaxies :

$$\left|\frac{\delta\alpha}{\alpha}\right| \leq 4\times10^{-12} \text{ / light year}$$

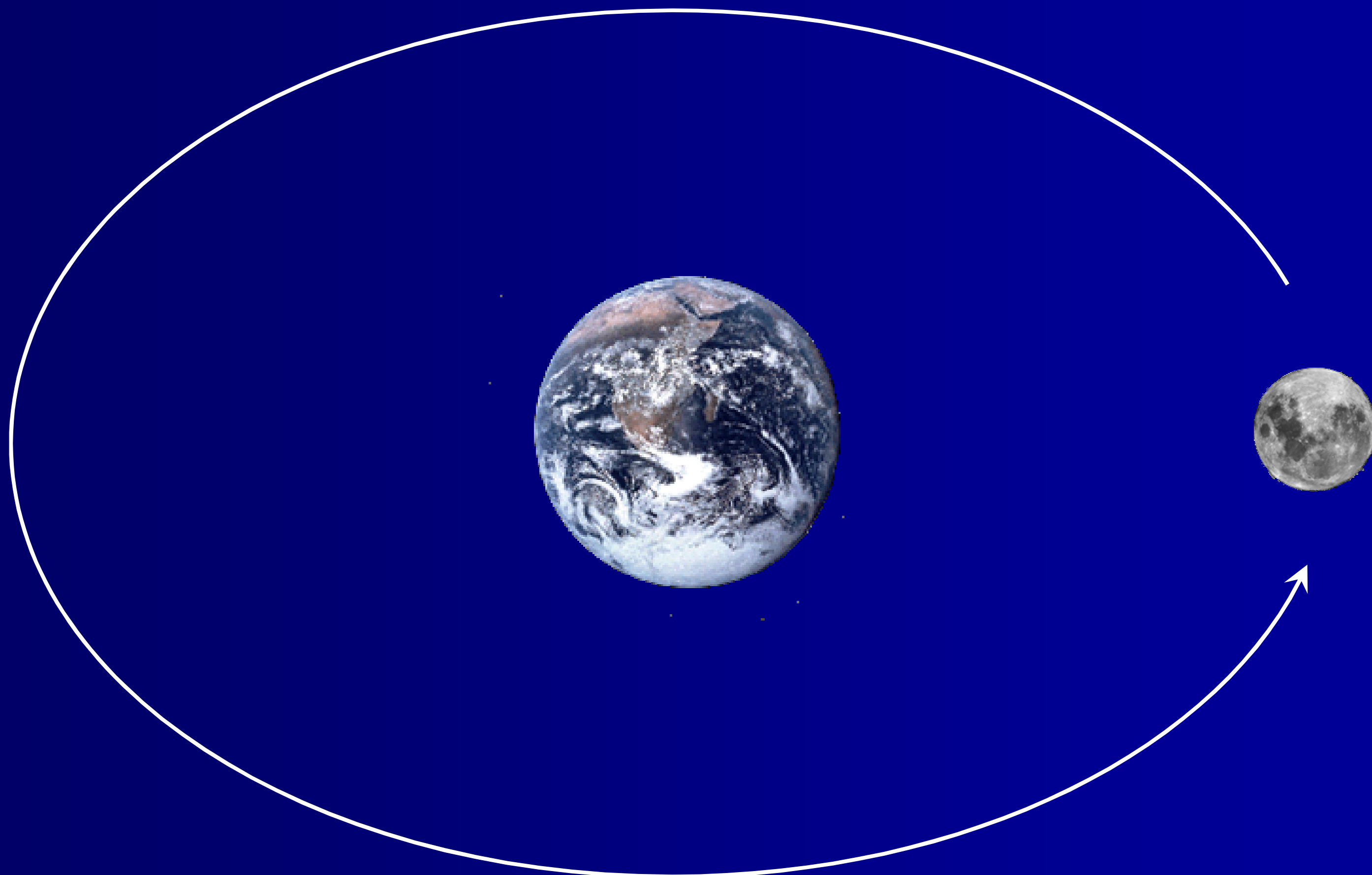
α really seems to be constant ; it does not vary logarithmically with the age of the universe.

Newton’s constant: Earth – Moon laser ranging:

Does Earth-Moon distance change with time?



Suppose that angular momentum stays constant,
But gravitational constant weakens ?



Then measuring the distance using atomic clocks means (after little calculation) that one measures

$$\frac{G\, m_P^3}{\hbar\, c\, m_e} = 1.085 \times 10^{-35}$$

It is found to change less than 1 part in 10^{12} per year. So, here also, Dirac cannot be right ...

$$\frac{m_P}{m_e} = 1836.1526726(8)$$

Proton/electron mass ratio from quasar measurements:

$$\left| \frac{\delta(m_P / m_e)}{m_P / m_e} \right| \leq 3 \times 10^{-15} \text{ / year}$$

The anthropic argument:

"Constants of Nature are what they are (very big or very small) because if they weren't, we wouldn't be here to observe them ..."

Other universes, with different constants, exist but are not inhabited ...

One version of this anthropic argument is obviously true:

There are many other planets, but none of them have the conditions such that they can be inhabited by life such as ours ...

Important difference however:

Planets can be *enumerated*, but constants of nature are *real numbers*, they are *not* denumerable.

Where are those other universes ??

What does superstring theory *have to say ?*

String theory was originally thought to harbor the solution to every problem, including this one.

The constants of nature were thought to be unique.

The string equations are unique, but the spacetime they live in is not ,

$$R(3,1) \otimes CY(6)$$

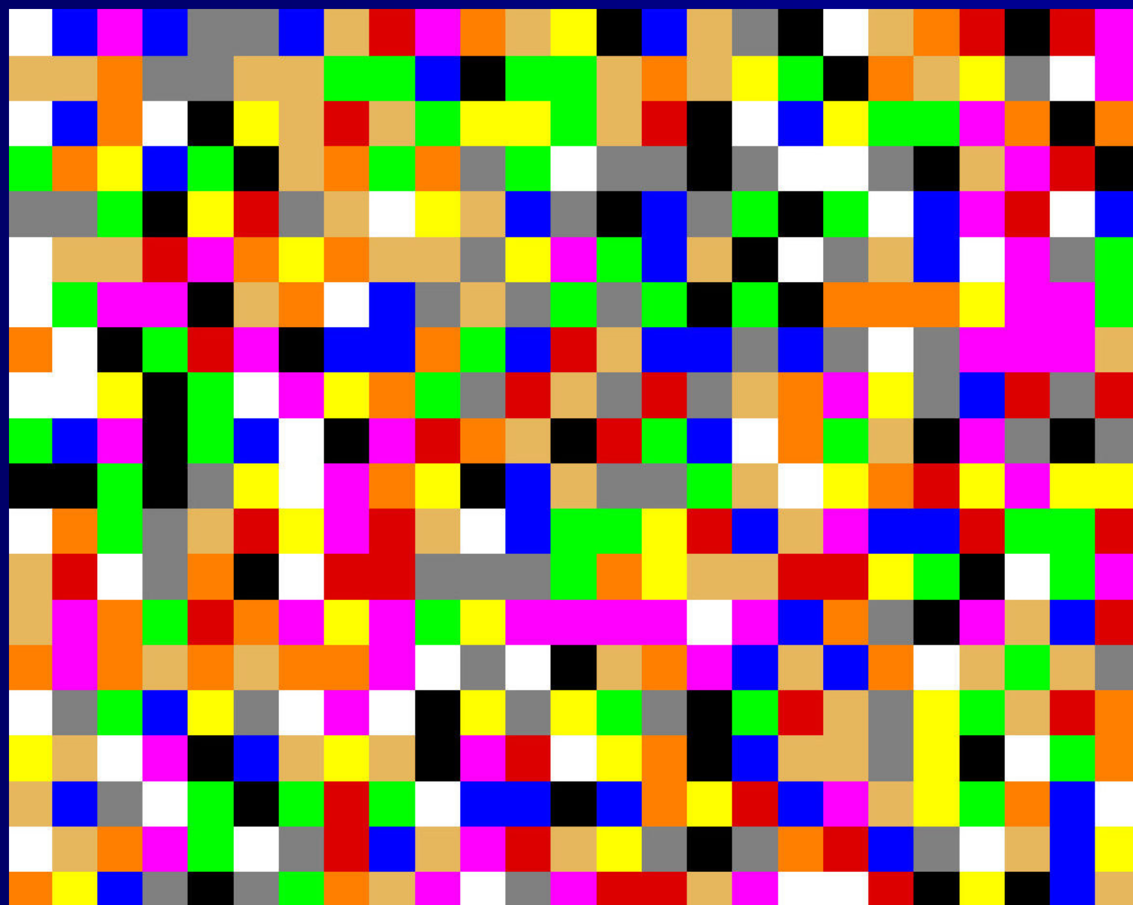
$CY(6)$ is a compactified space, but its topological shape – boundary conditions – can be one out numerous possibilities.

Then, this space can be filled with lower dimensional structures (D-branes) and flux quanta of many types of string-generated fields. These branes and fluxes are exactly conserved quantum numbers.

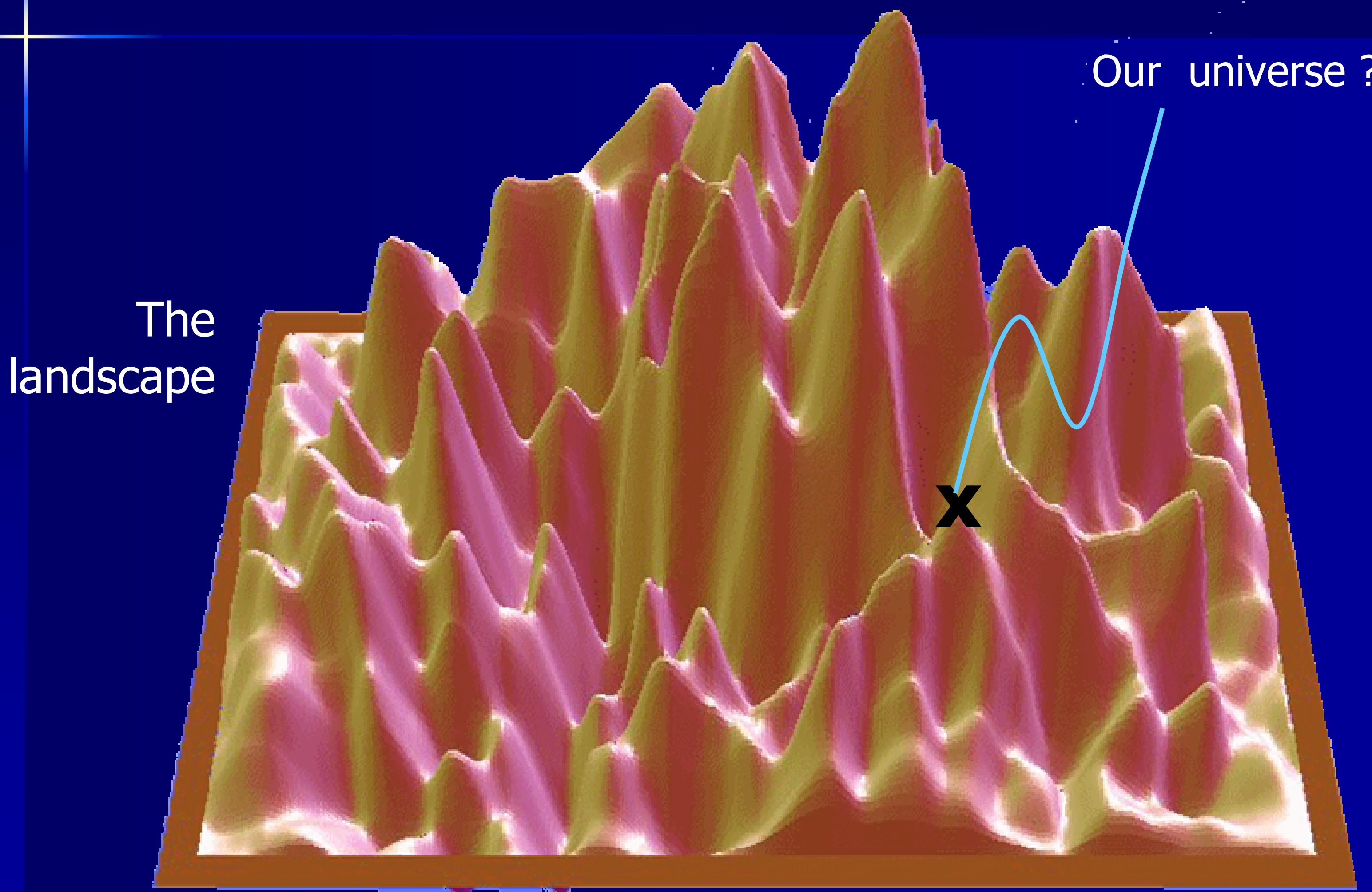
The observed "constants of Nature" depend on them.

The number of possibilities is unlimited, but denumerable. It has been estimated that 10^{500} is a realistic number …

It is not very large, but larger than comfortable:



Do all these 10^{500} universes “hang together” ?
Are they separated by phase boundaries ?



However, studying a simple model of quantum gravity with matter gives us more hope.

At the Planck scale, *Quantum Mechanics* and *General Relativity* must be unified into one.

But that is not all.

The physical laws of **black holes** must fit in this theory in a logical way as well.

You don't need string theory to note that this gives further constraints: an outside observer and an inside observer should agree about what happens

black hole complementarity

Observer entering a black hole

Observer watching from a distance

The theory of black hole complementarity seems to work only if all interaction parameters obey equations that we can determine …

But, there is a problem:

All constants will be *of order 1* in Planck units …

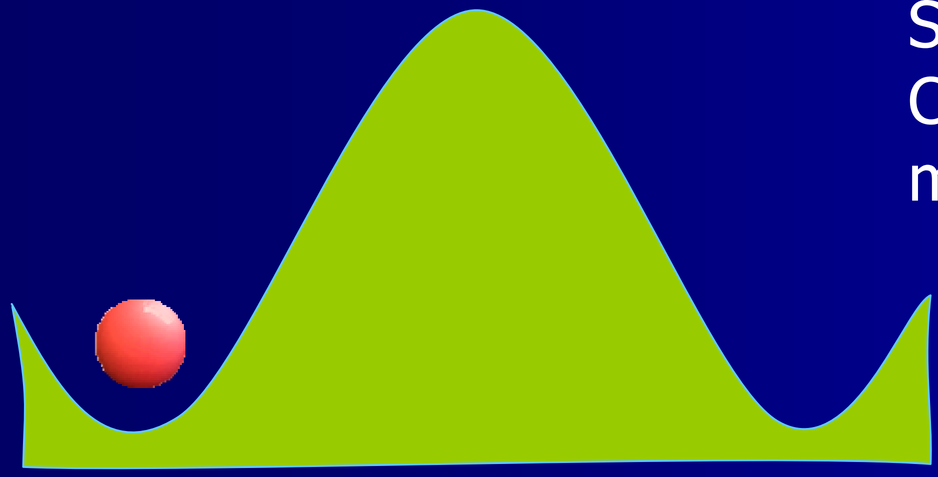
In this theory, the Universe is bound to stay very tiny …

Not all hope is lost.

Our equations depend on the symmetry structures of the particles we put in.

If a particle can have 10 different states, we can arrange the system such that its coupling strength is 1/100 .

Then some couplings arise as a tunneling effect:



Small probability:
Coupling strengths may result in

$$e^{-100}$$

Our universe may well be complex enough to generate large numbers all by itself !

Our model must be chosen in a very special way.

The End