Leptogenesis: in your face!

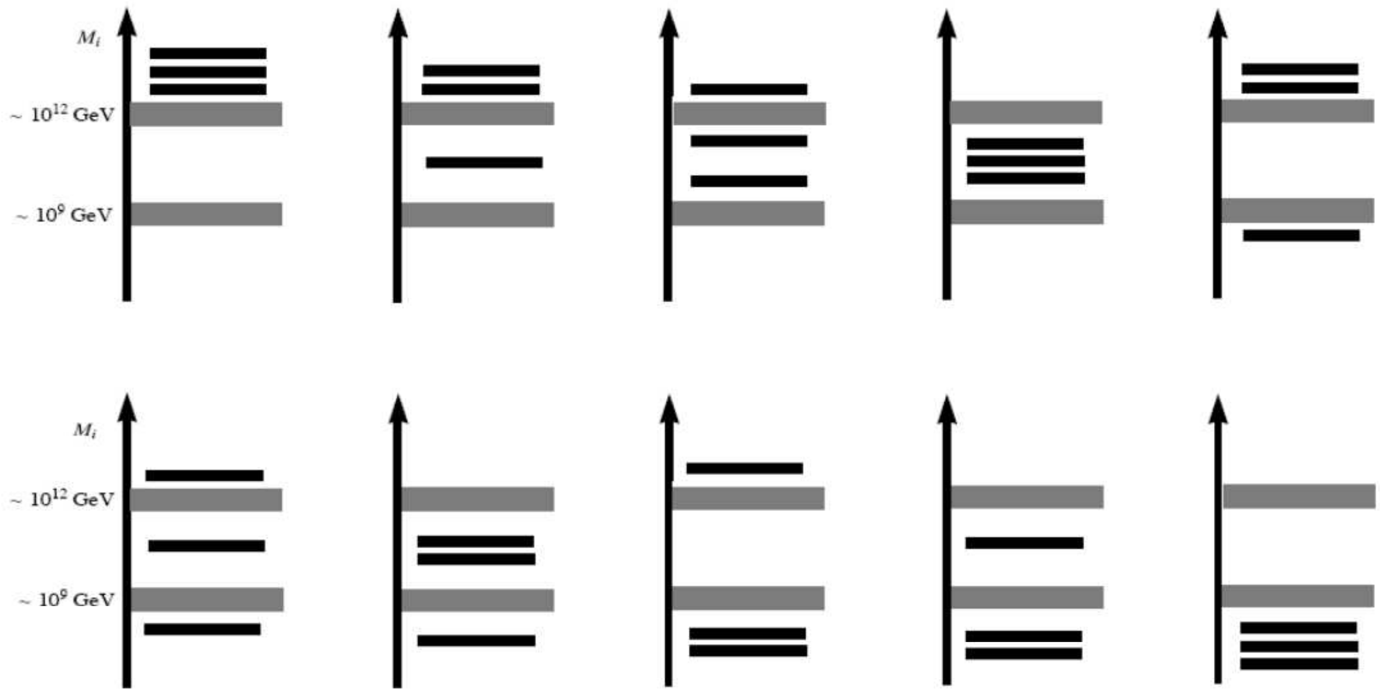
Part I: Decoherence

Part II: Our recent paper (arxiv:1112.4528)

My title is in homage to **Sidney Coleman's** lecture "Quantum Mechanics in your face" (1994). Leptogenesis encourages us to confront some fundamental aspects of quantum mechanics.

David A. Jones, University of Southampton, Invisibles ITN pre-meeting, 30/3/12

Leptogenesis - The Boltzmann Picture:



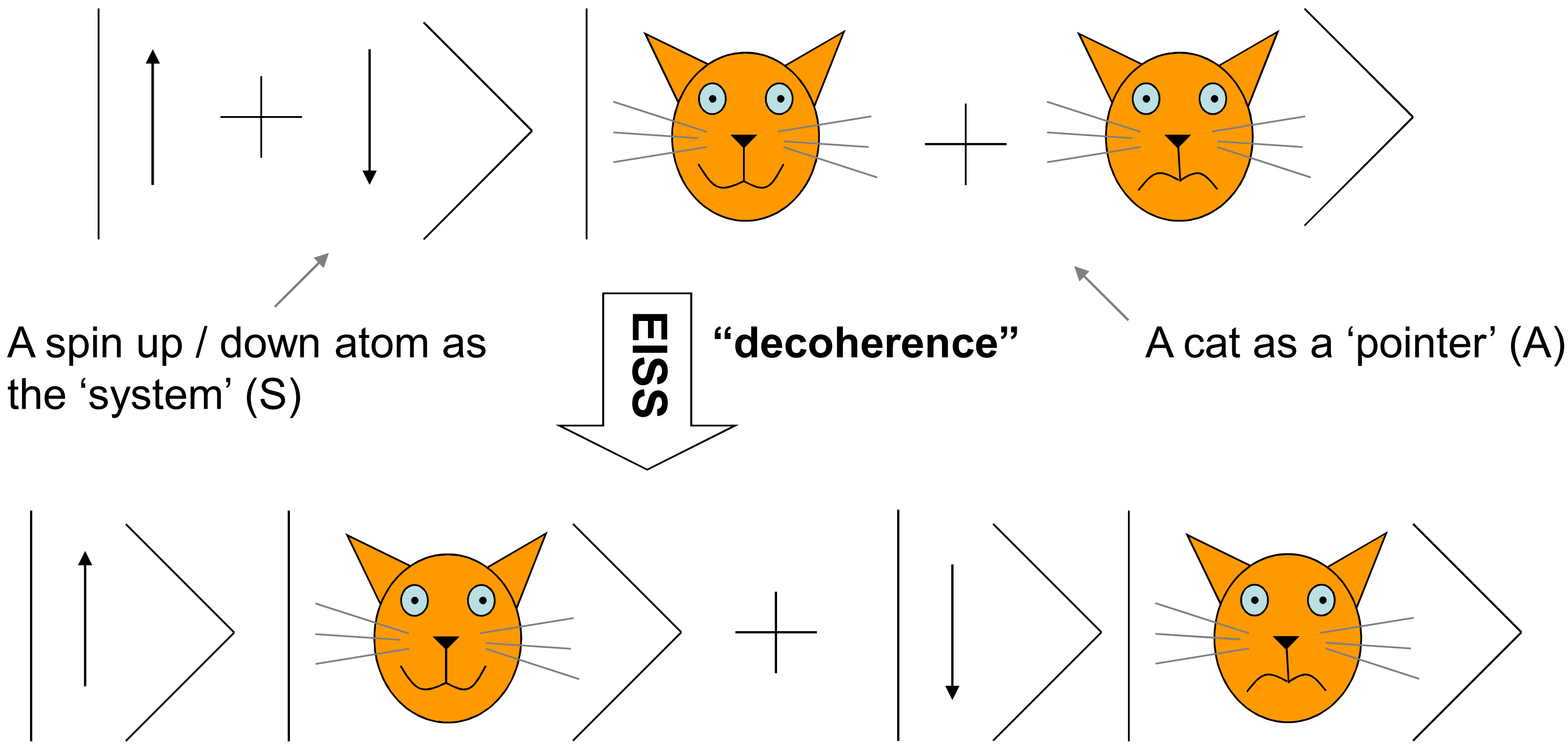
Each of the above mass-patterns is described by a different set of Boltzmann Equations (BE). These are derived somewhat heuristically…

Also, the BE don't describe the transition regimes – in grey.

We want to go beyond Boltzmann equations, to find a single density matrix equation, describing any allowed spectrum of RH neutrino masses. This means describing **decoherence**…

Decoherence via “Environment Induced Super-Selection”:

Quantum states of the ‘environment’ (E) get entangled with quantum states of the ‘system’ + ‘apparatus ‘ (SA). These interactions erase cross-correlations between ‘system’ (S) and ‘apparatus’ (A) states:



The Density Matrix

'System' + 'Environment' quantum state $\longrightarrow$ $$|\psi_{SE}\rangle = |S\rangle\,|E\rangle$$

Density Matrix $\longrightarrow$ $$\rho = |\psi_{SE}\rangle\,\langle\psi_{SE}|$$

Reduced Density Matrix $\longrightarrow$ $$\rho_r = Tr_E\,|\psi_{SE}\rangle\,\langle\psi_{SE}|$$

The reduced density matrix describes a "pre-measurement" state.

Subsequent entanglement of $|\psi_{SE}\rangle$ with apparatus $|A\rangle$ results in a measurement:

$$\left\langle \hat{\mathcal{O}}_S \right\rangle = Tr\left[\rho_r \hat{\mathcal{O}}_S\right]$$

NOTE: The state $|\psi_{SAE}\rangle$ has a preferred basis due to the **tridecompositional uniqueness theorem.** In this 'pointer' basis, decoherence (via **EISS**) rapidly erases the off-diagonal terms of the reduced density matrix

A "toy model" of decoherence:

The 'system' (S) is a spin up / down atom.

The 'environment' (E) is N other atoms, each in an undetermined state.

$$\widehat{H}_{\mathcal{S}\mathcal{E}} = (|\Uparrow\rangle\langle\Uparrow| - |\Downarrow\rangle\langle\Downarrow|) \otimes \sum_k g_k (|\uparrow_k\rangle\langle\uparrow_k| - |\downarrow_k\rangle\langle\downarrow_k|) \bigotimes_{k'\neq k} \widehat{I}_{k'}$$

S-E coupling

$$|\psi(t)\rangle = a|\Uparrow\rangle|E_\Uparrow(t)\rangle + b|\Downarrow\rangle|E_\Downarrow(t)\rangle$$

$$|E_\Uparrow(t)\rangle = |E_\Downarrow(-t)\rangle = \bigotimes_{k=1}^{N} (\alpha_k e^{ig_k t}|\uparrow_k\rangle + \beta_k e^{-ig_k t}|\downarrow_k\rangle)$$

The reduced density matrix $\rho_{\mathcal{S}}(t) = \mathrm{Tr}_{\mathcal{E}}(|\psi(t)\rangle\langle\psi(t)|)$ is then:

$$\rho_{\mathcal{S}}(t) = |a|^2|\Uparrow\rangle\langle\Uparrow| + |b|^2|\Downarrow\rangle\langle\Downarrow| + z(t)ab^*|\Uparrow\rangle\langle\Downarrow| + z^*(t)a^*b|\Downarrow\rangle\langle\Uparrow|$$

$$\langle|z(t)|^2\rangle_{t\to\infty} \simeq 2^{-N}\prod_{k=1}^{N}[1 + (|\alpha_k|^2 - |\beta_k|^2)^2] \xrightarrow{N\to\infty} 0 \longleftarrow$$

So E kills the off-diagonal terms for large t, N

Originally introduced by Zurek, see: Zurek, W.H.,1982, Phys. Rev. D **26**, 1982

For a nice, recent review on decoherence see: Schlosshauer, M., arxiv:quant-ph/0312059

Leptogenesis and decoherence:

In leptogenesis the decays of RH nus produce coherent states.

Subsequently, these decohere via coupling to thermal bath states.

Leptogenesis vs. "toy model" :

Quantity	**Toy model**	**Leptogenesis**
Environment (E)	N atoms, of undetermined spin	Thermal bath of SM quantum states
System (S)	Coherent state: $\vert S\rangle = (a\vert\Uparrow\rangle + b\vert\Downarrow\rangle)$	Coherent state: $\vert l_i\rangle = \sum_\alpha \vert\alpha\rangle\langle\alpha\vert l_i\rangle$
S-E coupling	g_k (k=1,..,N)	Neutrino Yukawas: $h_{\alpha i}$
Decoherence:	$\begin{pmatrix} \vert a\vert^2 & a\,b^* \\ a^*\,b & \vert b\vert^2 \end{pmatrix} \rightarrow \begin{pmatrix} \vert a\vert^2 & 0 \\ 0 & \vert b\vert^2 \end{pmatrix}$	$\begin{pmatrix} N^{B-L}_{\alpha\alpha} & N^{B-L}_{\alpha\beta} \\ N^{B-L\,*}_{\alpha\beta} & N^{B-L}_{\beta\beta} \end{pmatrix} \rightarrow \begin{pmatrix} N^{B-L}_{\alpha\alpha} & 0 \\ 0 & N^{B-L}_{\beta\beta} \end{pmatrix}$


INTERMISSION

Part II: The Paper

Leptogenesis with heavy neutrino flavours: from density matrix to Boltzmann equations

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Abstract

Leptogenesis with heavy neutrino flavours is discussed within a density matrix formalism. We write the density matrix equation that describes the generation of the matter-antimatter asymmetry, for an arbitrary choice of the right-handed (RH) neutrino masses. For hierarchical RH neutrino masses lying in the fully flavoured regimes, the density matrix equation reduces to multiple-stage Boltzmann equations. In this case we recover and extend results previously derived within a quantum state collapse description. We confirm the generic existence of phantom terms, which are not washed out at production and contribute to the flavoured asymmetries proportionally to the initial RH neutrino abundances. Even in the N_1-dominated scenario they can give rise to lepton flavour asymmetries much larger than the baryon asymmetry with potential applications. We also confirm that there is a (orthogonal) component in the asymmetry produced by the heavier RH neutrinos which completely escapes the washout from the lighter RH neutrinos and show that phantom terms additionally contribute to it. The other (parallel) component is washed out with the usual exponential factor, even for a weak washout. Finally, as an illustration, we study the two RH neutrino model in the light of the above findings, showing that phantom terms can contribute in this case as well.

What we did:

- We took into account decoherence, using the density matrix formalism.
- We derived a ‘Master Equation’ which can be used to calculate Baryon asymmetry within any transitional regime, between two definite flavour bases.
- This approach goes beyond Boltzmann Equations, which pre-suppose some definite flavour basis (whether “two-flavour”, “three-flavoured” or “unflavoured” / “vanilla”).
- We obtain the Boltzmann Picture from the density matrix formalism, as limiting cases of our ‘Master Equation’. We show how to derive **the projection effect & phantom leptogenesis** from our Master Equation.

Our Master Equation:

$$\begin{aligned}\frac{dN^{B-L}_{\alpha\beta}}{dz} &= \varepsilon^{(1)}_{\alpha\beta} D_1 (N_{N_1} - N^{\rm eq}_{N_1}) - \frac{1}{2} W_1 \{\mathcal{P}^{0(1)}, N^{B-L}\}_{\alpha\beta} \\ &+ \varepsilon^{(2)}_{\alpha\beta} D_2 (N_{N_2} - N^{\rm eq}_{N_2}) - \frac{1}{2} W_2 \{\mathcal{P}^{0(2)}, N^{B-L}\}_{\alpha\beta} \\ &+ \varepsilon^{(3)}_{\alpha\beta} D_3 (N_{N_3} - N^{\rm eq}_{N_3}) - \frac{1}{2} W_3 \{\mathcal{P}^{0(3)}, N^{B-L}\}_{\alpha\beta}\end{aligned}$$

$$\mathrm{i}\,\mathrm{Re}(\Lambda_\tau) \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar\ell} \right]_{\alpha\beta} - \mathrm{Im}(\Lambda_\tau) \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta}$$

$$\mathrm{i}\,\mathrm{Re}(\Lambda_\mu) \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar\ell} \right]_{\alpha\beta} - \mathrm{Im}(\Lambda_\mu) \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta}$$

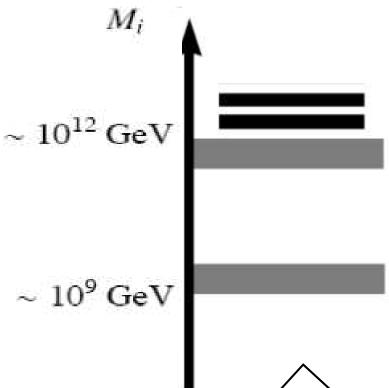
$$N^{l+\bar l} \equiv N_{l+\bar l}\,\rho^{l+\bar l}$$
$$N^{B-L} \equiv N_{B-L}\,\rho^{B-L}$$
$$\mathcal{P}^{0(i)}_{\alpha\beta} \equiv \frac{h^*_{\alpha i} h_{\beta i}}{(h^\dagger h)_{ii}}$$

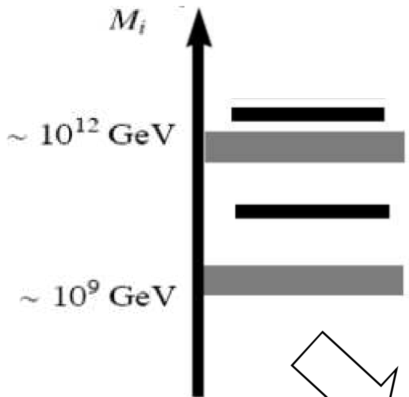
Flavour Oscillations

Decoherence / Flavour Damping

We didn't solve the above explicitly in the transitional regime (this is hard!) Instead we took **limiting cases**, taking into account both **projection effects** and **phantom terms** . But the full equations above are now available, for ourselves or others to solve in future ☺

Our limiting cases:





Projectors:

$$p^0_{xy} \equiv \frac{1}{2}\left(|\langle x|y\rangle|^2 + |\langle x|y\rangle|^2\right)$$

$$\Delta p_{xy} \equiv |\langle x|y\rangle|^2 - |\langle x|y\rangle|^2$$

Section 3.1

“3 Stages **Phantom Leptogenesis**”

$$N^{\rm f}_{B-L} \simeq p^0_{2\alpha}\, N^{T\simeq T_{B2}}_{B-L} - \frac{\Delta p_{2\alpha}}{2}\, N^{\rm in}_{N_2}$$

Section 3.2

“2 Stages **Phantom Leptogenesis**”

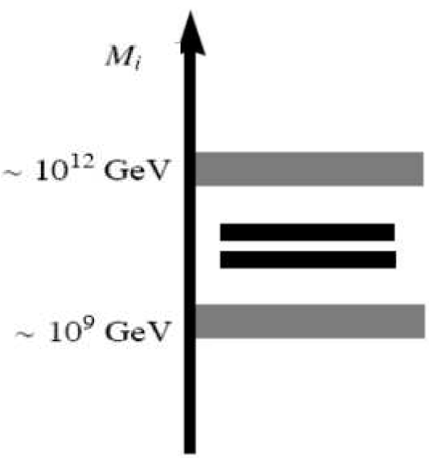
$$N^{\rm f}_{B-L} = \varepsilon_1\,\kappa(K_1) + \left[p^0_{12}\, e^{-\frac{3\pi}{8}\,K_1} + (1 - p^0_{12})\right]\,\varepsilon_2\,\kappa(K_2) + \left(1 - e^{-\frac{3\pi}{8}\,K_1}\right)\,\frac{\Delta p_{12}}{2}\,N^{\rm in}_{N_2}$$

Projection effect

Phantom terms

Section 4.1

“Two Right-Handed Neutrino Model”



$$\begin{aligned} N^{\rm f}_{B-L} &= \varepsilon_{1\tau}\,\kappa(K_{1\tau}) + \varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}} \\ &+ \varepsilon_{1\tau_1^\perp}\,\kappa(K_{1\tau_1^\perp}) + \left(p^0_{\tau_1^\perp\tau_2^\perp}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) - \frac{\Delta p_{\tau_1^\perp\tau_2^\perp}}{2}\,N^{\rm in}_{N_2}\right)\,e^{-\frac{3\pi}{8}\,K_{1\tau_1^\perp}} \\ &+ \left(1 - p^0_{\tau_1^\perp\tau_2^\perp}\right)\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \frac{\Delta p_{\tau_1^\perp\tau_2^\perp}}{2}\,N^{\rm in}_{N_2}\,, \end{aligned}$$

Projection effect and phantom terms:

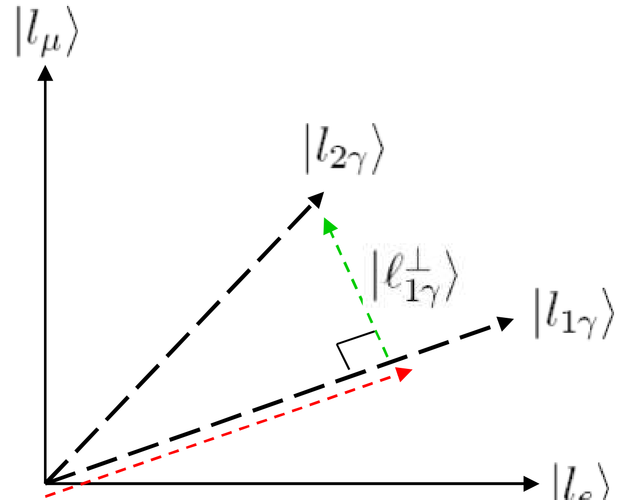
Projection effect:

What if N_1 inverse decays act on coherent states, for example $\gamma = e + \mu$?

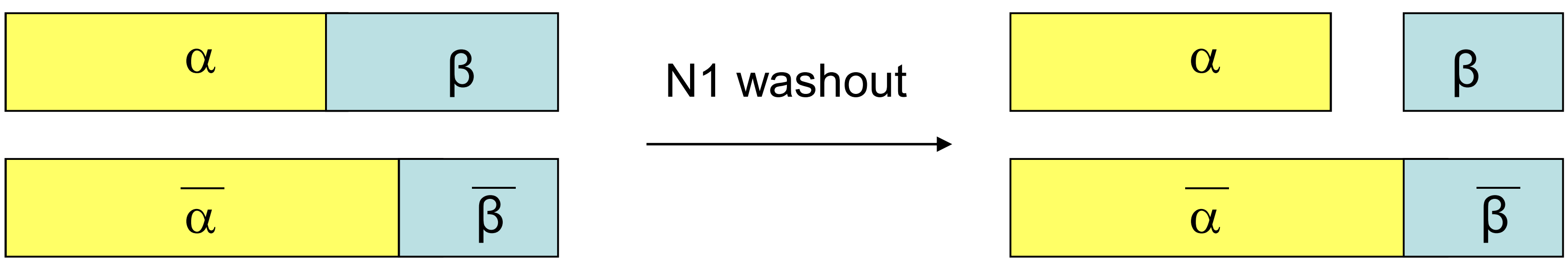
Then part of the coherent state produced by N_2 is **orthogonal** to the coherent state washed out by N_1

Only the **parallel** component can be washed out by N_1 inverse decays.

The **orthogonal** part is **completely unwashed**.



Phantom terms:



What if quantum states produced by N2 have different lepton, anti-lepton flavour compositions? Then cancellations within the flavoured asymmetry can be **erased by an asymmetric N1 washout** (if l1,l2 belong to distinct flavour bases).

Summary / Conclusions:

- “The Decoherence Program” helps to formulate a quantum theory of measurement.
- Decoherence is relevant within leptogenesis scenarios.
- In our recent paper, we gave the full density matrix equations for leptogenesis, incorporating decoherence, hence describing the **transitional regimes**.
- We derived the Boltzmann equations for some limiting cases of the full density matrix equations. We showed how the projection effect and phantom terms can be derived within a density matrix formalism.