An SU(5) GUT with non-zero θ_{13} from A4

A4$\times$SU(5) GUT of Flavour with Trimaximal Neutrino Mixing, Cooper, King and Luhn, arXiv:1203.1324

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Outline

Introduction

- Various experiments have measured neutrino oscillation and mass differences.
- Recently, the third oscillation angle, θ_{13} has been observed to be non-zero
- Exciting! Potential to measure CP violation in oscillation experiments
- Also points towards TB mixing only being part of the story (since this predicts $\theta_{13} = 0$)
- $$\Delta m^2_{21}[10^{-5} eV^2] = 7.59^{0.20}_{-0.18} \quad \Delta m^2_{31}[10^{-3} eV^2] = 2.5^{0.09}_{-0.16}$$
  $$\sin^2\theta_{12} = 0.312^{0.017}_{-0.015} \quad \sin^2\theta_{23} = 0.52^{0.06}_{-0.07} \quad \sin^2\theta_{13} = 0.092^{0.017}_{-0.017}$$

The Type I seesaw

- Introduce a heavy Majorana particle ν_R

$$\mathcal{L}_N = -\left(L_L^T \widetilde{H} Y_\nu \nu_R^* + h.c.\right) - \frac{1}{2}\left(\left(\nu_R^c\right)^T M_{RR} \nu_R + h.c.\right)$$

- ν_R can be integrated out below M_{RR} to give an effective theory
- The Lagrangian now looks like a Majorana mass for the ν_L

$$\mathcal{L}_\nu = -\frac{1}{2}\nu_L^T m_{LL} \nu_L + h.c.$$

$$m_{LL} \sim -v^2 Y_\nu M_{RR}^{-1} Y_\nu^T = -m_{LR} M_{RR}^{-1} m_{LR}^T \text{ with } v = \langle H \rangle$$

- This is equivalent to diagonalising the 6×6 matrix

$$\begin{pmatrix} 0 & \left(m_{LR}^D\right)^T \\ m_{LR}^D & M_{RR} \end{pmatrix} \longrightarrow \begin{pmatrix} m_{LL} & 0 \\ 0 & M_{RR} \end{pmatrix} \quad + \quad \sout{\textit{higher order terms}}$$

under the assumption that $M_{RR} \gg m_{LR}$

Trimaximal mixing

- Before T2K, Double Chooz, Daya Bay, oscillation data was consistent with the choice

$$\theta_{12} = \sin^{-1}\frac{1}{\sqrt{3}},\ \theta_{13} = 0,\ \theta_{23} = \frac{\pi}{4} \Rightarrow U_{PMNS} = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- This is the tribimaximal mixing pattern
- A combination of:
  - Trimaximal mixing in the ν_2 eigenstate
  - And bimaximal mixing in the ν_3 eigenstate
- Now there are observations ($\sim 5.2\sigma$) of a sizeable θ_{13}, with a central value $\sim 8.8^\circ$.

Trimaximal mixing

- In light of this evidence, new mixing patterns must be considered.
- Trimaximal mixing is promising: it retains the trimaximally mixed second column whilst allowing for $\theta_{13} \neq 0$
- $$U_{\mathrm{TM}}^{\nu} = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\vartheta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\vartheta\, e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\vartheta - \frac{1}{\sqrt{2}}\sin\vartheta\, e^{-i\rho} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\cos\vartheta - \frac{1}{\sqrt{6}}\sin\vartheta\, e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\vartheta + \frac{1}{\sqrt{2}}\sin\vartheta\, e^{-i\rho} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\cos\vartheta - \frac{1}{\sqrt{6}}\sin\vartheta\, e^{i\rho} \end{pmatrix}$$
- Here, $\frac{2}{\sqrt{6}}\sin\vartheta = \sin\theta_{13}^{\nu}$
- In fact we can recover TB mixing by taking $\theta_{13} \to 0$; maybe this suggests we can take a flavour model giving TB mixing and adapt it to obtain TM mixing?

The Atarelli Feruglio model

- A very simple and well known model of TB neutrino mixing is based on A_4, the even permutation group on four elements (or the group of permutations of a tetrahedron)
- Has a triplet representations (where we unify our left handed lepton fields), **3**, and three inequivalent one dimensional representations, **1**, **1**$'$ and **1**$''$
- Multiplication of triplets is as follows:

$$\mathbf{3} \otimes \mathbf{3} \sim \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3_S} + \mathbf{3_A}$$

  and in particular, the singlet above is constructed as follows (basis dependent)

$$a_1 b_1 + a_2 b_3 + a_3 b_2$$

- The fact that the product of two triplets contains a triplet means that we can make a singlet with any number of triplets > 1.
- Singlets multiply with addition of primes modulo 3.

The Altarelli Feruglio model

- Neutrino sector:

$$w_\nu = yh(Nl) + \left(x_A\xi + \widetilde{x}_A\xi'\right)(NN) + x_B\left(\varphi_S NN\right)$$

  Here we have combined the flavours of lepton into a triplet, l, and similarly for right handed neutrinos, N.

- φ_S is a flavon - scalar triplet of A_4 which obtains a VEV and leads to the flavour structure

$$\langle\varphi_S\rangle = v_S\begin{pmatrix}1\\1\\1\end{pmatrix}$$

- ξ and ξ' are singlet flavons of A_4 with VEVs u and 0 respectively. All VEVs are complex
- The ξ contributes to reproducing the TB structure, while ξ' is required for the VEV alignment of φ_S

The Altarelli Feruglio model

- Once the flavons obtain their VEVs, Dirac and Majorana masses are given as

$$m_D = y v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad M_{RR} = \begin{pmatrix} A + \frac{2B}{3} & -\frac{B}{3} & -\frac{B}{3} \\ -\frac{B}{3} & \frac{2B}{3} & A - \frac{B}{3} \\ -\frac{B}{3} & A - \frac{B}{3} & \frac{2B}{3} \end{pmatrix} u$$

$A = 2x_A$ and $B = 2x_B \frac{v_S}{u}$

- This leads to an effective LH neutrino mass matrix diagonalised by U_{TB} and with eigenvalues

$$m_1 = \frac{y^2}{(A+B)} \frac{v_u^2}{u} \quad m_2 = \frac{y^2}{A} \frac{v_u^2}{u} \quad m_3 = \frac{y^2}{(-A+B)} \frac{v_u^2}{u}$$

TM mixing from A4

- This model can be extended very minimally to predict trimaximal mixing (after all it is essentially a perturbation to TB).
- Introduce flavon singlet in the $\mathbf{1}''$ representation - this means there are now flavon singlets furnishing each one dimensional A4 representation.
- Neutrino sector now looks like

$$w_\nu = yh(Nl) + \left(y_1\varphi_S + y_2\xi + y_3'\xi' + y_3''\xi''\right)(NN)$$

which now leads to

$$m_D = yv_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_{RR} = \left[\alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \gamma'' \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

TM as a perturbation from TB

- The above matrix may be rewritten as a sum of two matrices, one of which preserves TB mixing and one which violates it:

$$M_{RR} = M_{RR}^{TB} + \Delta M_{RR}$$

$$M_{RR}^{TB} = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\Delta M_{RR} = \Delta \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

- Here $\Delta = \frac{1}{2}(\gamma'' - \gamma')$ and $\gamma = \frac{1}{2}(\gamma' + \gamma'')$
- Since experimentally the mixing is still close to TB mixing, we must have $|\Delta| \ll |\alpha|, |\beta|$, wheras no such constraint applies to γ.

TM as a perturbation from TB

- This observation allows one to diagonalise M_{RR} perturbatively, such that one ends up with $U_{TM} = U_{TB} + \Delta U$
- Performing this procedure gives the lepton mixing matrix arising from the A4 model

$$U_{TM} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \end{pmatrix}$$

- This is of trimaximal form, and is manifestly a perturbation from TB mixing.
- The complex parameter α_{13} is the only combination of input parameters (i.e. α, β, γ, Δ) which appears.

$$Re\left(\alpha_{13}\right) = -\frac{\sqrt{3}}{2}\left[Re\left(\frac{\Delta}{\beta-\gamma}\right) + Im\left(\frac{\Delta}{\beta-\gamma}\right)\frac{Im\left(\frac{3\alpha}{\beta-\gamma}\right)}{Re\left(\frac{3\alpha}{\beta-\gamma}\right)}\right]$$

$$Im\left(\alpha_{13}\right) = \frac{\sqrt{3}}{2}\frac{Im\left(\frac{3\alpha}{\beta-\gamma}\right)}{Re\left(\frac{3\alpha}{\beta-\gamma}\right)}$$

TM as a perturbation from TB

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$$U_{TM} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \end{pmatrix}$$

- This is of trimaximal form, and is manifestly a perturbation from TB mixing. In order to try and constrain α_{13}, one can use TB deviation parameters:

$$U_{dev} \approx \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s), \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a), \quad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

TM as a perturbation from TB

$$U_{TM} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \end{pmatrix}$$

$$U_{dev} \approx \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

- Comparing the two matrices, one may read off

$$s \approx 0, \quad a \approx \frac{\mathrm{Re}\,(\alpha_{13})}{\sqrt{3}}, \quad r\cos\delta \approx -\frac{2}{\sqrt{3}}\mathrm{Re}\,(\alpha_{13})\,, \quad \delta \approx \arg\alpha_{13} + \pi$$

$$\Rightarrow a \approx -\frac{1}{2} r\cos\delta$$

  independently of α_{13}

- Once we uplift to a GUT, we will see these rules receive Cabibbo size corrections

$A4 \otimes SU(5)$

Field	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	$H_{\overline{45}}$
$SU(5)$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\overline{\mathbf{45}}$
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$
$U(1)_R$	1	1	1	1	1	0	0	0
$U(1)$	1	-1	3	3	0	0	-1	-2
Z_2	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$-$
Z_3	ω	ω^2	ω^2	1	1	1	ω	ω
Z_5	ρ	ρ^4	1	1	1	1	ρ	ρ

Table: Matter and Higgs chiral superfields in the model.

A4⊗$SU(5)$

Field	φ_S	ξ	ξ'	ξ''	φ_T	θ	θ'	θ''	$\widetilde{\theta}'$	σ
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$
$U(1)_R$	0	0	0	0	0	0	0	0	0	0
$U(1)$	-2	-2	-2	-2	2	-1	-1	-1	-5	2
Z_2	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$+$
Z_3	ω	ω	ω	ω	1	ω	ω^2	ω^2	ω^2	1
Z_5	ρ^3	ρ^3	ρ^3	ρ^3	1	1	1	1	1	1

Table: Flavon chiral superfields in the model.

LO terms - neutrinos

$$W_\nu = yFNH_{\mathbf{5}} + \left(y_1\varphi_S + y_2\xi + y_3'\xi' + y_3''\xi''\right) NN$$

- This is the simplest part - it's the same as before.
- Mass matrices are

$$m_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u$$

and

$$M_R = \left[\alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \gamma'' \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

LO terms - down quarks/charged leptons

$$W_d \sim \left(\frac{\theta^2\theta''}{\Lambda_d^4} (F\varphi_T)' + \frac{\theta^2\theta'}{\Lambda_d^4} (F\varphi_T)'' \right) H_{\overline{\mathbf{5}}} T_1 + \frac{\sigma\theta\theta' (\theta'')^2}{\Lambda^5} (F\varphi_T) H_{\overline{\mathbf{45}}} T_1$$
$$+ \frac{(\theta')^2 \theta''}{\Lambda_d^4} (F\varphi_T) H_{\overline{\mathbf{5}}} T_2 + \left(\frac{\theta\theta''}{\Lambda_d^3} (F\varphi_T)' + \frac{\theta\theta'}{\Lambda_d^3} (F\varphi_T)'' \right) H_{\overline{\mathbf{45}}} T_2$$
$$+ \left(\frac{\sigma^2\theta^2 (\theta')^2}{\Lambda^6} (F\varphi_T) + \frac{1}{\Lambda_d} ((F\varphi_T)'') \right) H_{\overline{\mathbf{5}}} T_3 + \left(\frac{\sigma^2\theta^3}{\Lambda^5} (F\varphi_T)' \right) H_{\overline{\mathbf{45}}} T_3$$

- When we let the flavons and Higgs fields obtain their VEVs, this results in

$$\begin{pmatrix} k_f \eta_\sigma \eta_\theta \eta_{\theta'} \eta_{\theta''}^2 & \eta_\theta^2 \eta_{\theta''} & \eta_\theta^2 \eta_{\theta'} \\ \eta_{\theta'}^2 \eta_{\theta''} & k_f \eta_\theta \eta_{\theta''} & k_f \eta_\theta \eta_{\theta'} \\ \eta_\sigma^2 \eta_\theta^2 \eta_{\theta'}^2 & k_f \eta_\sigma^2 \eta_\theta^3 & 1 \end{pmatrix} \eta_T v_d$$

- Here, $\eta_i = \frac{\langle|\varphi_i|\rangle}{\Lambda}$ is a small parameter. Λ can differ between sectors (so it will be different in the up quark sector)
- Charged lepton and down quark matrices unified $\Rightarrow$ we need some way of ensuring different masses - k_f from the $H_{\overline{\mathbf{45}}}$ does this for us. It is 1 for the down quark matrix and -3 for the charged leptons

LO terms - down quarks

$$M_d \sim \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^6 & \epsilon^5 & 1 \end{pmatrix} \epsilon\, v_d \quad M_e \sim \begin{pmatrix} -3\epsilon^5 & \epsilon^3 & \epsilon^6 \\ \epsilon^3 & -3\epsilon^2 & -3\epsilon^5 \\ \epsilon^3 & -3\epsilon^2 & 1 \end{pmatrix} \epsilon\, v_d$$

- Here we assume the numerical value $\epsilon \sim 0.15$.
- These give mass ratios of $\epsilon^4 : \epsilon^2 : 1$ for the down quarks and $\frac{\epsilon^4}{3} : 3\epsilon^2 : 1$ for the charged leptons
- In the low quark angle approximation, left-handed down quark mixing angles $\theta^d_{12} \sim \epsilon$, $\theta^d_{13} \sim \epsilon^3$ and $\theta^d_{23} \sim \epsilon^2$ are also predicted in agreement with data (assuming an approximately diagonal up sector which we obtain in the next Section).
- The corresponding charged lepton mixing angles are $\theta^e_{12} \sim \frac{\epsilon}{3}$, $\theta^e_{13} \sim \epsilon^6$ and $\theta^e_{23} \sim 3\epsilon^5$.

LO terms - up quarks

$$W_u \sim \frac{\theta^4 (\theta')^2}{\Lambda_u^6} T_1 T_1 H_{\mathbf{5}} + \left(\frac{\theta^2 (\theta')^2 (\theta'')^2}{\Lambda_u^6}\right) (T_1 T_2 + T_2 T_1) H_{\mathbf{5}} + \frac{\theta^2 \theta'}{\Lambda_u^3} (T_1 T_3 + T_3 T_1) H_{\mathbf{5}}$$
$$+ \frac{\theta \widetilde{\theta'}}{\Lambda_u^2} T_2 T_2 H_{\mathbf{5}} + \frac{\theta' (\theta'')^2}{\Lambda_u^3} (T_2 T_3 + T_3 T_2) H_{\mathbf{5}} + T_3 T_3 H_{\mathbf{5}}.$$

- In terms of the parameter η_i defined previously, this matrix may be written as

$$\begin{pmatrix} \eta_\theta^4 \eta_{\theta'}^2 & \eta_\theta^2 \eta_{\theta'}^2 \eta_{\theta''}^2 & \eta_\theta^2 \eta_{\theta'} \\ \eta_\theta^2 \eta_{\theta'}^2 \eta_{\theta''}^2 & \eta_\theta \eta_{\widetilde{\theta}} & \eta_{\theta'} \eta_{\theta''}^2 \\ \eta_\theta^2 \eta_{\theta'} & \eta_{\theta'} \eta_{\theta''}^2 & 1 \end{pmatrix} v_u$$

- The top mass is renormalisable here

LO terms - up quarks

$$M_u \sim \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^6 & \bar{\epsilon}^3 \\ \bar{\epsilon}^6 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^3 & 1 \end{pmatrix} v_u$$

- This gives the up quark mass hierarchy $\bar{\epsilon}^6 : \bar{\epsilon}^3 : 1$
- This matrix is approximately diagonal, meaning the CKM mixing arises predominantly from the down quark sector, with the Cabibbo angle being $\theta_C \sim \theta_{12}^d \sim \epsilon (\Rightarrow \theta_{12}^e \sim \frac{\theta_C}{3})$

CL corrections to neutrino mixing

- The presence of sizeable charged lepton mixing means that the physical mixing parameters are not simply from the neutrino sector, but are corrected by this lepton mixing
- The unification of charged leptons and down quarks means that this correction is in fact parameterised by the Cabibbo angle
- Incorporating these corrections leads to the corrected sum rule bounds

$$|s| \leq \frac{\theta_C}{3} \qquad |a| \leq \frac{1}{2}\left(r + \frac{\theta_C}{3}\right)|\cos\delta|$$

- Taking $\theta_{13} \sim 8.8^\circ$ gives $r \sim 0.22$; then using $\frac{\theta_C}{3} \sim 0.075$ gives the bounds

$$|s| \leq 0.075 \qquad |a| \leq 0.15\,|\cos\delta|$$

- These are consistent with approximate limits from the global fit of

$$-0.06 < s < 0 \qquad |a| < 0.08$$

Summary and Outlook

- Recent neutrino data points towards non-zero θ_{13}
- This means we need to move away from TB mixing - but not too far!
- One can extend a well known A4 model of TB mixing to predict TM mixing instead
- This results in a sum rule between mixing angles which can constrain the extra parameters of the model
- It is also possible to extend the model to a GUT, in order to try and generate the quark mixings as well
- Unified nature of the model introduces Cabibbo sized corrections into the neutrino mixings
- Translating these into bounds on TB deviation parameters is approximately consistent with data
- Would be nice to remove some of the extra flavons if possible