The Potential of Minimal Flavour Violation

Rodrigo Alonso

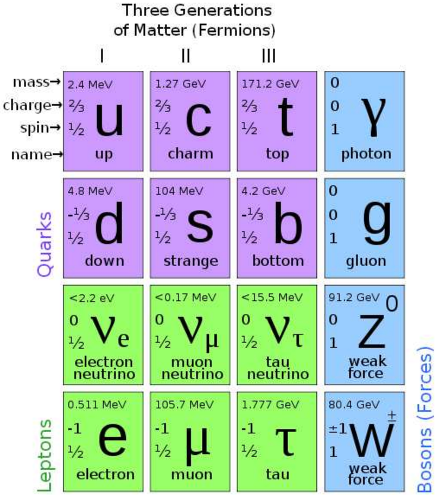
Invisibles Premeeting, Madrid, March 2012

Departamento de Física Teórica
Universidad Autónoma de Madrid
& IFT-UAM/CSIC

based on the work with Belén Gavela, Luca Merlo , Stefano Rigolin & Daniel Hernández JHEP **1107**, 012 (2011). arXiv:1103.2915.

Outline

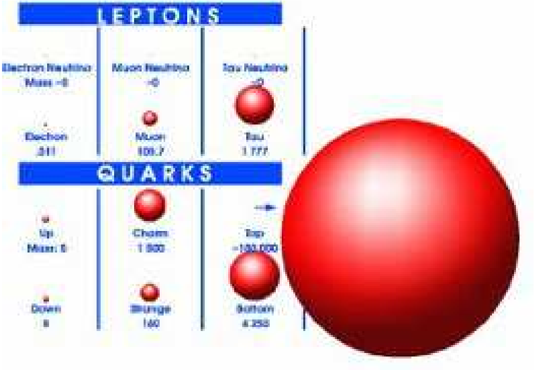
The Flavour Puzzle



- Why 3 generations? CP violation?
- Visible part of the universe $\rightarrow$ 1^{st} generation

The Three Generations

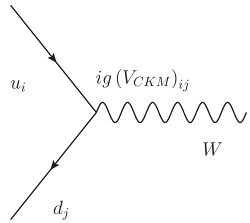
How do we tell one generation from the other?



A wildly hierarchical mass spectrum

The Three Generations

How do they connect with each other?



$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & ?0.2 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

The Flavour Puzzle

The flavour structure is encoded in the Yukawa couplings in the SM

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$-\mathcal{L}_{Yukawa} = \overline{Q}_L Y_U U_R \tilde{H} + \overline{Q}_L Y_D D_R H + \overline{\ell}_L Y_E E_R H + h.c. + (\nu \text{ mass})$$

That can be parametrized as

$$Y_U = V_{CKM}^{\dagger} \mathbf{y}_U \qquad Y_D = \mathbf{y}_D \qquad Y_E = \mathbf{y}_E$$

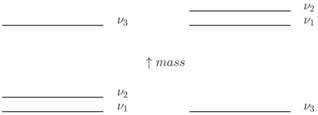
$$\& \;\; m_{\nu_1}\,,\; m_{\nu_2}\;\; m_{\nu_3}$$

$$\mathbf{y}_U \sim \text{Diag}\,(m_u, m_c, m_t)$$

$$\mathbf{y}_D \sim \text{Diag}\,(m_d, m_s, m_b)$$

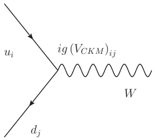
$$\mathbf{y}_E \sim \text{Diag}\,(m_e, m_\mu, m_\tau)$$

Normal or Inverted hierarchy

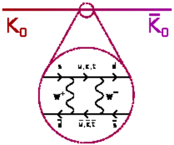


Flavour Physics

The Flavour Phenomenology of the SM is very specific



- There are no Flavour Changing Neutral Currents at tree level in the SM
- FCNC processes occur at the loop level and GIM suppressed
- CP violation 'small'; Jarlskog Inv.×GIM in SM

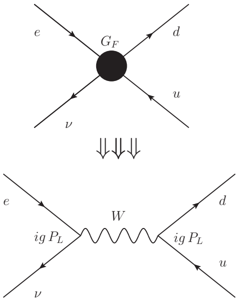


→ Sensitivity to new physics

Beyond the Standard Model; Effective Field Theory

Fermi's Theory of β decay

$$\mathcal{L}_{em} + G_F \bar{e}\gamma_\mu \nu \bar{u}\gamma^\mu d$$

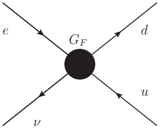


$$\mathcal{L}_{em} + (g^2/M_W^2)\bar{e}\gamma_\mu \nu_L \bar{u}\gamma^\mu d_L$$

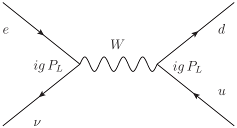
Beyond the Standard Model; Effective Field Theory

Fermi's Theory of β decay

$$\mathcal{L}_{em} + G_F \bar{e}\gamma_\mu \nu \bar{u}\gamma^\mu d$$



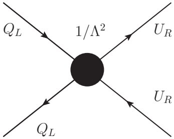
$$\Downarrow\Downarrow\Downarrow$$



$$\mathcal{L}_{em} + (g^2/M_W^2)\bar{e}\gamma_\mu \nu_L \bar{u}\gamma^\mu d_L$$

BSM physics

$$\mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{O}^{d=5} + \frac{1}{\Lambda^2}\mathcal{O}^{d=6} + \ldots$$



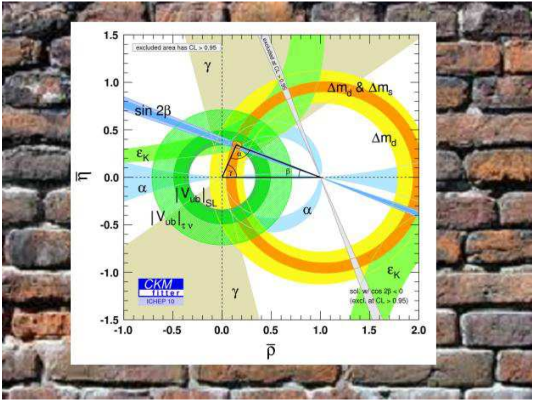
$$\Downarrow$$

????

which is a valid description up to Λ

The Flavour Problem

All the flavour data is $\sim$ consistent with the SM

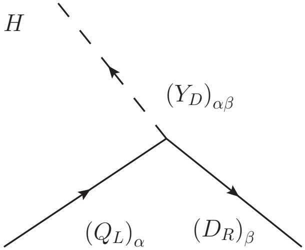


This translates into $\Lambda > 10^{2-3}\, TeV$

Minimal Flavour Violation (MFV)

MFV is a **symmetry approach** to the flavour problem that takes the **experimental data at face value**.

The MFV hypothesis: *The Yukawa couplings are the* ***only*** *sources of flavour violation in and* ***beyond*** *the Standard Model*[1].



[1] Georgi & Chivukula 1987; D'Ambrosio, Giudice, Isidori, & Strumia, 2002; Cirigliano, Grinstein, Isidori & Wise 2005.

Minimal Flavour Violation; Realization

- Generations are distinguished by masses; in the limit of zero mass

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \not{\mathcal{L}}_{Yukawa}$$

the SM presents an extended symmetry group :

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \overbrace{SU(3)_{\ell_L} \times SU(3)_{E_R}}^{Lepton} \times \cdots$$

$$D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad D_R \sim (1, 3, 1 \cdots)$$

- The Yukawa couplings break the symmetry, unless

$$\overline{Q}_L Y_D D_R H \qquad Y_D \sim (3, \bar{3}, 1) \quad \text{'SPURIONS'}$$

Minimal Flavour Violation; Realization

- Generations are distinguished by masses; in the limit of zero mass

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \not{\mathcal{L}}_{Yukawa}$$

the SM presents an extended symmetry group :

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \overbrace{SU(3)_{\ell_L} \times SU(3)_{E_R}}^{Lepton} \times \cdots$$

$$D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad D_R \sim (1, 3, 1 \cdots)$$

- The Yukawa couplings break the symmetry, unless

$$\overline{Q}_L Y_D D_R H \qquad Y_D \sim (3, \bar{3}, 1) \quad \text{'SPURIONS'}$$

MFV: Effective Field Theory Prescription

Take the operator

$$\mathcal{L}_{SM} + \frac{c_{\alpha\beta}}{\Lambda_{NP}^2} \overline{Q}_\alpha \gamma^\mu Q_\beta \overline{E}_R \gamma_\mu E_R + \ldots$$

containing all flavors α, β, then:

$$\text{MFV} \;\to\; c_{\alpha\beta} = (Y_U)_{\alpha\gamma} \left(Y_U^\dagger\right)_{\gamma\beta} \simeq V_{t\alpha}^* V_{t\beta} y_t^2$$

$$c \sim (3 \times \bar{3}, 1, 1) = (3, 1, \bar{3}) \times (\bar{3}, 1, 3) \sim (3 \times \bar{3}, 1, 1)$$

The predictions have the Standard Model flavour structure:

$$\mathcal{A}(d^i \to d^j)_{MFV} = (V_{ti}^* V_{tj}) \mathcal{A}_{SM}^{(\Delta F=1)} \left(1 + \frac{(4\pi)^2 y_t^2 M_W^2}{\Lambda_{NP}^2}\right)$$

Antonelli et al.

The strongest experimental bound sets $\Lambda_{NP} > \text{TeV} \sim \Lambda_{EW}$.

MFV: Effective Field Theory Prescription

Take the operator

$$\mathcal{L}_{SM} + \frac{c_{\alpha\beta}}{\Lambda_{NP}^2} \overline{Q}_\alpha \gamma^\mu Q_\beta \overline{E}_R \gamma_\mu E_R + \ldots$$

containing all flavors α , β, then:

$$\text{MFV} \;\rightarrow\; c_{\alpha\beta} = (Y_U)_{\alpha\gamma} \left(Y_U^\dagger\right)_{\gamma\beta} \simeq V_{t\alpha}^* V_{t\beta} y_t^2$$

$$c \sim (3 \times \bar{3}, 1, 1) = (3, 1, \bar{3}) \times (\bar{3}, 1, 3) \sim (3 \times \bar{3}, 1, 1)$$

The predictions have the Standard Model flavour structure:

$$\mathcal{A}(d^i \rightarrow d^j)_{MFV} = (V_{ti}^* V_{tj}) \mathcal{A}_{SM}^{(\Delta F=1)} \left(1 + \frac{(4\pi)^2 y_t^2 M_W^2}{\Lambda_{NP}^2}\right)$$

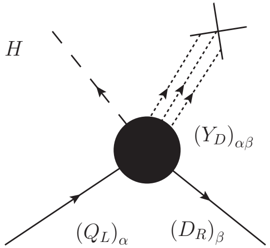
Antonelli et al.

The strongest experimental bound sets $\Lambda_{NP} > \text{TeV} \sim \Lambda_{EW}$.

What if we take seriously the flavour symmetry reasoning?

The Dynamics Behind MFV

Suggests that the Yukawa couplings have a *dynamical origin* .



The Yukawa Interaction involves extra fields increasing the dimension of the 'Yukawa Operator'.

$Y \sim \langle \Psi \rangle$? $\quad Y = \left\langle \Psi^2 \right\rangle$? $\quad Y = \langle \Psi^n \rangle$?

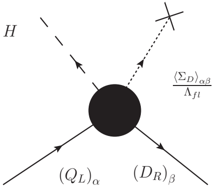
These fields acquire a v.e.v. and fix the Yukawa couplings.

The Dynamics Behind MFV: Quarks

Different cases shorted by EFT & Group Theory

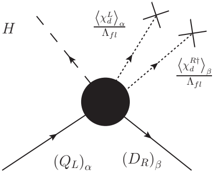
1. The Yukawas are the vev of 1 field
   $Y \sim \langle \Sigma \rangle$
   - Dim. 5 $\leftrightarrow$ Bifundamental Fields

$$\overline{Q}_L \frac{\Sigma_d}{\Lambda_f} D_R H \qquad \Sigma_d \sim (3, \bar{3}, 1)$$



2. The Yukawas are 'composite' $Y \sim \langle \chi\chi \rangle$
   - Dim. 6 $\leftrightarrow$ Fundamental Fields

$$\overline{Q}_L \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} D_R H \qquad \chi_d^L \sim (3, 1, 1) \qquad \chi_d^R \sim (1, 3, 1)$$



What is the potential giving rise to spontaneous symmetry breaking?

What are the differences in this potential for Quarks and Leptons?

Let's start with quarks and the fundamental field case for illustration

What is the potential giving rise to spontaneous symmetry breaking?

What are the differences in this potential for Quarks and Leptons?

Let's start with quarks and the fundamental field case for illustration

What is the potential giving rise to spontaneous symmetry breaking?

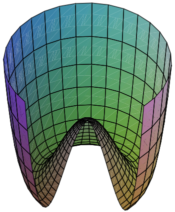
What are the differences in this potential for Quarks and Leptons?

Let's start with quarks and the fundamental field case for illustration

Fundamental Fields: Quarks

- First gather invariants

$$\chi_u^{L\dagger}\chi_u^L\,, \qquad \chi_u^{R\dagger}\chi_u^R\,, \qquad \chi_d^{L\dagger}\chi_d^L\,,$$
$$\chi_d^{R\dagger}\chi_d^R\,, \qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right|\left|\chi_d^L\right|\cos\theta_c\,.$$



- Put the invariants together in the potential:

$$V' = \sum_{i=u,d}\lambda_i\left(\chi_i^{L\dagger}\chi_i^L - \frac{\mu_i^2}{2\lambda_i}\right)^2 + \lambda_{ud}\left(\chi_u^{L\dagger}\chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}}\right)^2 + \cdots$$

Minimum (2 gen.): $y_c^2 = \frac{\mu_u^2}{2\lambda_u\Lambda^2}\,, \quad y_s^2 = \frac{\mu_d^2}{2\lambda_d\Lambda^2}\,, \quad \cos\theta = \frac{\mu_{ud}^2\sqrt{\lambda_u\lambda_d}}{\mu_u\mu_d\lambda_{ud}}$

θ naturally O(1)

Fundamental Fields (are special)

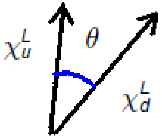
Masses: Let's take a careful look at the Yukawa structure:

$$Y_D = \frac{\left\langle \chi_d^L \chi_d^{R\dagger} \right\rangle}{\Lambda^2} \quad ; \quad Y_U = \frac{\left\langle \chi_u^L \chi_u^{R\dagger} \right\rangle}{\Lambda^2}$$

A 'matrix' made out of 2 'vectors '.

Such a construction has rank 1 , that is
one nonvanishing eigenvalue only!

Mixing:



Diagonalizing the Yukawa coupling =
finding the moduli ∼ masses +
the relative angle ∼ mixing

Fundamental Fields (are special)

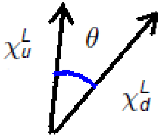
Masses: Let's take a careful look at the Yukawa structure:

$$Y_D = \frac{\left\langle \chi_d^L \chi_d^{R\dagger} \right\rangle}{\Lambda^2} \quad ; \quad Y_U = \frac{\left\langle \chi_u^L \chi_u^{R\dagger} \right\rangle}{\Lambda^2}$$

A 'matrix' made out of 2 'vectors '.

Such a construction has rank 1 , that is
one nonvanishing eigenvalue only!

Mixing:



Diagonalizing the Yukawa coupling =
finding the moduli $\sim$ masses +
the relative angle $\sim$ mixing

Bifundamental Fields

Invariants

$$\langle \Sigma_u \rangle = \Lambda_f \cdot V_{CKM}^{\dagger} \text{Diag}\{y_{u_i}\} , \qquad \langle \Sigma_d \rangle = \Lambda_f \cdot \text{Diag}\{y_{d_i}\} \; ;$$

$$\text{Tr}\left(\Sigma_u \Sigma_u^{\dagger} \Sigma_d \Sigma_d^{\dagger}\right) \propto \text{Tr}\left(V^{\dagger} y_U^2 V y_D^2\right) \xrightarrow{\text{2gen}} (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

The Potential:

$$V = -\mu_u^2 \text{Tr}\left(\Sigma_u \Sigma_u^{\dagger}\right) + \lambda_u \left(\text{Tr}\left(\Sigma_u \Sigma_u^{\dagger}\right)\right)^2 + \cdots$$

We can directly minimize in the 'vevs' $y_{d_i}, y_{u_i}, V_{CKM}$. Outcome:

- For obtaining the actual masses fine tunning is needed mass hierarchy
- For fixing the mixing huge (10^{-8}) fine tuning + dim≥ 8 Operators

Bifundamental Fields

Invariants

$$\langle \Sigma_u \rangle = \Lambda_f \cdot V^\dagger_{CKM} \text{Diag}\{y_{u_i}\}\,, \qquad \langle \Sigma_d \rangle = \Lambda_f \cdot \text{Diag}\{y_{d_i}\}\ ;$$

$$\text{Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) \propto \text{Tr}\left(V^\dagger y_U^2 V y_D^2\right) \xrightarrow{\text{2gen}} (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

The Potential:

$$V = -\mu_u^2 \text{Tr}\left(\Sigma_u \Sigma_u^\dagger\right) + \lambda_u \left(\text{Tr}\left(\Sigma_u \Sigma_u^\dagger\right)\right)^2 + \cdots$$

We can directly minimize in the 'vevs' $y_{d_i}, y_{u_i}, V_{CKM}$. Outcome:

- For obtaining the actual masses fine tunning is needed mass hierarchy
- For fixing the mixing huge (10^{-8}) fine tuning + dim≥ 8 Operators

What about Leptons?

What about Leptons?

- Dirac Mass ν's $\to$ All the previous analysis applies...
- Majorana Mass ν's
  - Weinberg operator

$$O_W = \frac{1}{M}\bar{\ell}_{L\alpha}\tilde{H}\lambda_{\alpha\beta}\tilde{H}^\dagger \ell^c_{L\beta}$$

$$\lambda \sim (3\times 3, 1) \qquad \text{under} \qquad SU(3)_{\ell_L} SU(3)_{E_R}$$

- Bifundamental
  $\lambda = \Sigma_\nu/\Lambda \sim U m_\nu U^T$
  $\to$Much like in the quark case $\Sigma_\nu\Sigma_\nu^\dagger \sim \Sigma_u\Sigma_u^\dagger$
- Fundamental
  $\lambda = \chi_\nu^2/\Lambda^2 \sim U m_\nu U^T$
  But this forces a 1 ν mass only...

Leptons

Lets go to a (predictive) Seesaw Model

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{l}_L H Y_E E_R + \bar{l}_L \tilde{H} Y N + \bar{l}_L Y' \tilde{H} N' + M \overline{N'} N^c + h.c. ,$$

$$m_\nu = \frac{v^2}{M} \left(Y\, Y'^T + Y'\, Y^T \right) \qquad |Y'| << |Y| \text{ approx LN}$$

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}} U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}} \\ \sqrt{m_{\nu_3}} \end{pmatrix}$$

Gavela, Hambye, P. Hernández, D. Hernández

The symmetry for vanishigh Yukawa couplings is: $SU(3)_{\ell_L} \times SU(3)_{E_R} \times U_{LN}$
The straightforward assignments:

$$Y_E = \frac{\langle \Sigma_E \rangle}{\Lambda} \sim (3\,,\,\bar{3})_0\,; \quad Y = \frac{\langle \chi_N \rangle}{\Lambda} \sim (3\,,\,1)_0\,; \quad Y' = \frac{\langle \chi'_N \rangle}{\Lambda} \sim (3\,,\,1)_2\,. \tag{1}$$

Leptons

The invariants containing the angle (2 family case & $(|Y'| << |Y|)$):

$$\chi_N^\dagger \Sigma_E \Sigma_E^\dagger \chi_N \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1})\cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}}\sin 2\alpha \sin 2\theta\right),$$

$[\chi_N^\dagger \Sigma_E \Sigma_E^\dagger \chi_N] = 4 \rightarrow$ Renormalizable level: $\partial_\theta V = 0$ yields:

$$-(m_{\nu_2} - m_{\nu_1})\sin 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}}\sin 2\alpha \cos 2\theta = 0 \rightarrow \quad \mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}.$$

Quarks

Let's bring back the quark invariant (Bifundamental):

$$\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

which yields

$$(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0 \rightarrow \quad \sin 2\theta = 0$$

Leptons

The invariants containing the angle (2 family case & $(|Y'| << |Y|)$):

$$\chi_N^\dagger \Sigma_E \Sigma_E^\dagger \chi_N \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \sin 2\theta\right) ,$$

$[\chi_N^\dagger \Sigma_E \Sigma_E^\dagger \chi_N] = 4 \rightarrow$ Renormalizable level: $\partial_\theta V = 0$ yields:

$$-(m_{\nu_2} - m_{\nu_1}) \sin 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \cos 2\theta = 0 \rightarrow \quad \mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} .$$

Quarks

Let's bring back the quark invariant (Bifundamental):

$$\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

which yields

$$(m_c^2 - m_u^2)(m_s^2 - m_d^2) \sin 2\theta = 0 \rightarrow \qquad \sin 2\theta = 0$$

Summary

The study of a SSB mechanism for flavour lead to

1. The flavour symmetry is restrictive, making the fixing of flavour parameters far from trivial.
2. The Fundamental case for Quarks gives a 'reason' for the strong hierarchical mass spectrum and accommodates mixing
3. The Lepton sector, when the Seesaw mechanism generates majorana masses, offers an interesting mass degeneracy $\Leftrightarrow$ maximal angles connection