Theoretical Models for Neutrino Masses

Luca Merlo

"Invisibles" pre-meeting

Madrid, 30 March 2012

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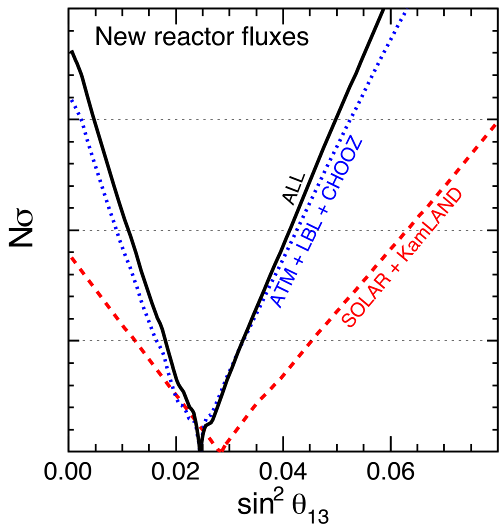
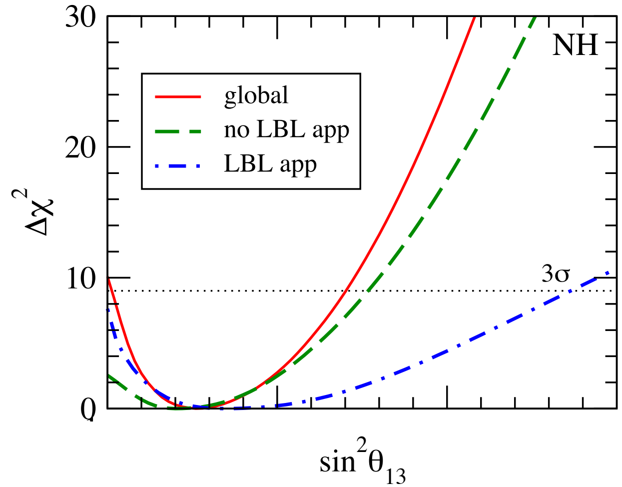
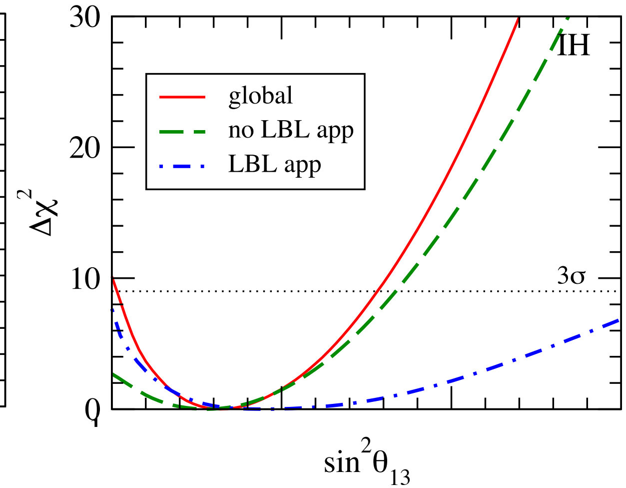
Recent Results of Global Fits

Assuming **3 light active oscillating** neutrinos:

with both T2K and MINOS data and new reactor fluxes







[Fogli *et al.* 2011]

$$\Delta m^2_{\rm sol} = (7.58^{+0.22}_{-0.26}) \times 10^{-5}\,{\rm eV}^2$$

$$\Delta m^2_{\rm atm} = (2.35^{+0.12}_{-0.09}) \times 10^{-3}\,{\rm eV}^2$$

$$\sin^2\theta_{12} = 0.312^{+0.017}_{-0.016}$$

$$\sin^2\theta_{23} = 0.42^{+0.08}_{-0.03}$$

$$\sin^2\theta_{13} = 0.025^{+0.007}_{-0.007}$$

[Schwetz *et al.* 2011]

$$\Delta m^2_{\rm sol} = (7.59^{+0.20}_{-0.18}) \times 10^{-5}\,{\rm eV}^2$$

$$\Delta m^2_{\rm atm} = (2.50^{+0.09}_{-0.16})[(2.40^{+0.08}_{-0.09})] \times 10^{-3}\,{\rm eV}^2$$

$$\sin^2\theta_{12} = 0.312^{+0.017}_{-0.015}$$

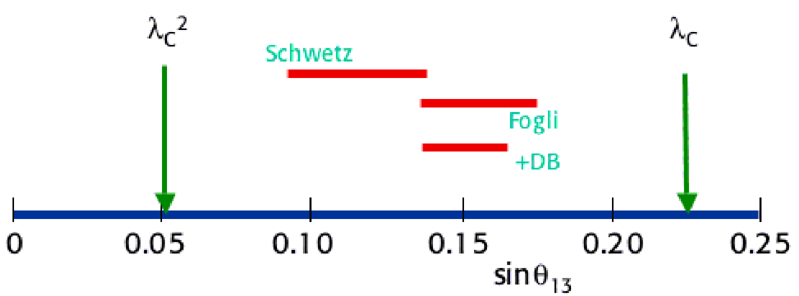
$$\sin^2\theta_{23} = 0.52^{+0.06}_{-0.07}[0.52 \pm 0.06]$$

$$\sin^2\theta_{13} = 0.013^{+0.007}_{-0.005}[0.016^{+0.008}_{-0.006}]$$

Recent Results on θ_{13}

	$\sin^2 2\theta_{13}$	$\sin^2 \theta_{13}$
T2K [1106.2822]	$0.11^{+0.11}_{-0.05}$ $(0.14^{+0.12}_{-0.06})$	$0.028^{+0.019}_{-0.024}$ $(0.036^{+0.022}_{-0.030})$
MINOS [1108.0015]	$0.041^{+0.047}_{-0.031}$ $(0.079^{+0.071}_{-0.053})$	$0.010^{+0.012}_{-0.008}$ $(0.020^{+0.019}_{-0.014})$
DC [1112.6353]	$0.086 \pm 0.041 \pm 0.030$	$0.022^{+0.019}_{-0.018}$
DYB [1203.1669]	$0.092 \pm 0.016 \pm 0.005$	0.024 ± 0.005

Our Average: $\sin^2 \theta_{13} = 0.022 \pm 0.004 (0.023 \pm 0.004)$



Neutrino Mass Patterns

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In the past:

- large atmospheric angle
- only upper bound on the reactor angle

$\longrightarrow$

$$\sin^2\theta_{23} = \frac{1}{2}$$

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mu-tau symmetry

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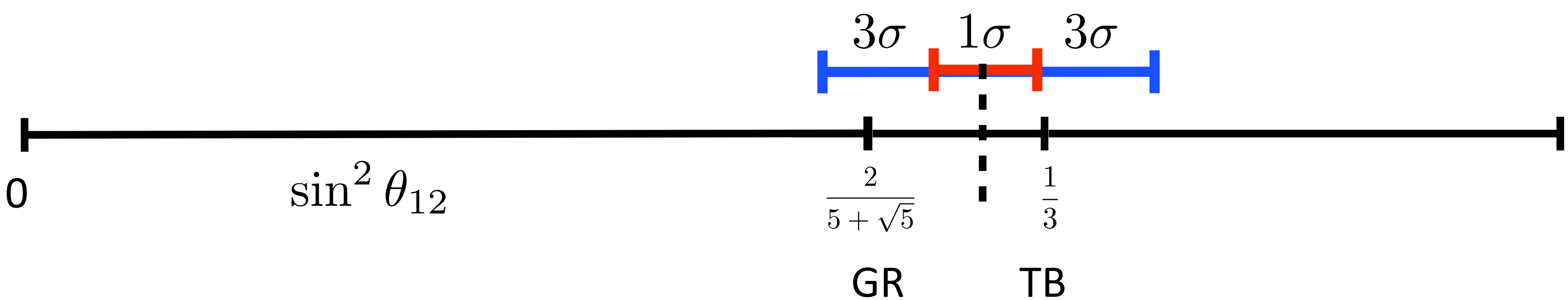


TRI-BIMAXIMAL (TB) [Harrison, Perkins & Scott 2002; Zhi-Zhong Xing 2002]

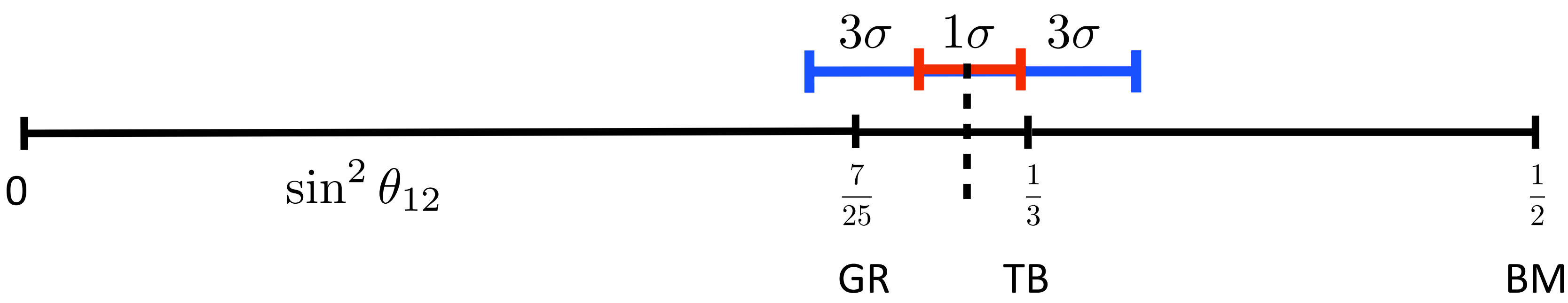
$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \sin^2\theta_{12} = \frac{1}{3} \longrightarrow \theta_{12} = 35.26^\circ$$

GOLDEN RATIO (GR) [Kajiyama, Raidal & Strumia 2007]

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \tan\theta_{12} = \frac{1}{\phi} \longrightarrow \theta_{12} = 31.72^\circ$$

$$\phi \equiv \frac{1+\sqrt{5}}{2}$$

Neutrino Mass Patterns



BIMAXIMAL (BM) [Vissani 1997; Barger et al. 1998]

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \sin^2\theta_{12} = \frac{1}{2} \quad \longrightarrow \quad \theta_{12} = 45^\circ$$

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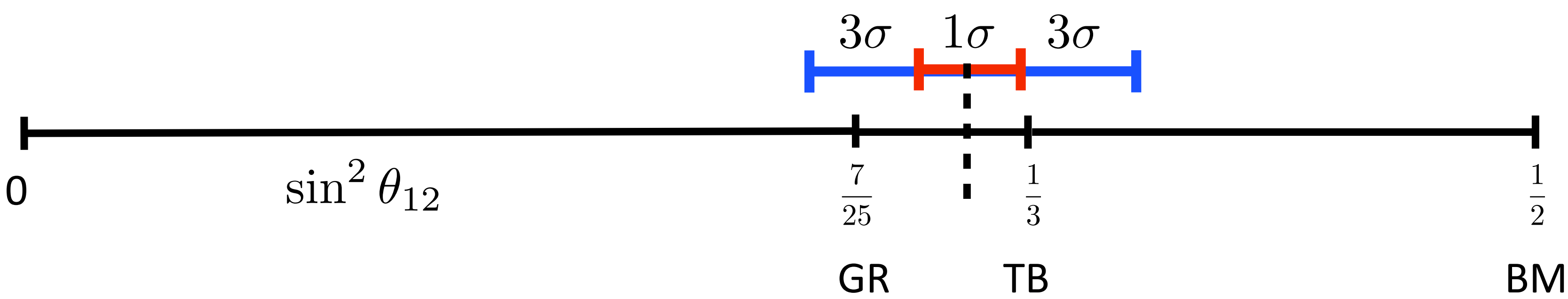

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Maybe related to the
Quark-Lepton Complementarity:
[Smirnov; Raidal; Minakata & Smirnov 2004]

$$\pi/4 \approx \theta_{12} + \lambda$$

$$\longrightarrow \quad \theta_{12}^{Exp} \approx \theta_{12}^{BM} - \lambda$$

[Altarelli, Feruglio and LM 2009, Adelhart, Bazzocchi and LM 2010, Meloni 2011]

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→ **Suitable** Spontaneous Symmetry Breaking $\varphi \to \langle\varphi\rangle$

→ at LO the PMNS can take one of the previous predictive patterns

→ at NLO, some corrections arise and they are proportional to the VEV of the flavons: larger is the VEV and larger are the corrections; loss of predictivity.

Typical Tri-Bimaximal

$$\sin^2\theta_{12}^{TB} = 1/3$$
$$\sin^2\theta_{23}^{TB} = 1/2$$
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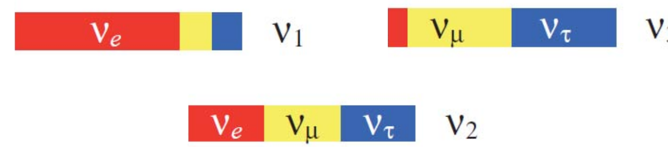

$$U^{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

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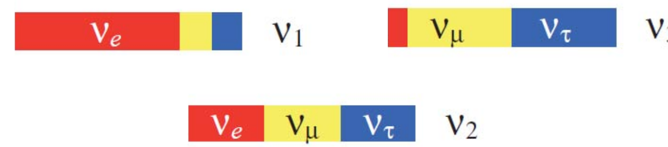

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In the basis of diagonal charged leptons:

$$M_\nu^{TB} = \begin{pmatrix} x & y & y \\ y & z & x+y-z \\ y & x+y-z & z \end{pmatrix}$$

mu-tau sym
magic sym

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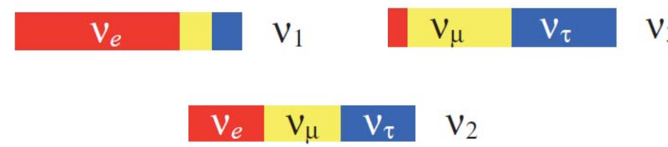

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Discrete Symmetries:

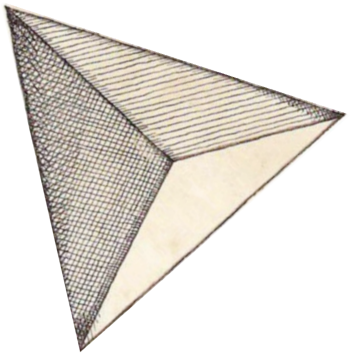
[A_4: Adhikary; Altarelli; Aristizabal Sierra; Babu; Bazzocchi; Bertuzzo; Di Bari; Branco; Brahmachari; Chen; Choubey; Ciafaloni; Csaki; Delaunay; Felipe; Feruglio; Frampton; Frigerio; Ghosal; Grimus; Grojean; Grossmann; Hagedorn; He; Hirsch; Honda; Joshipura; Kaneko; Keum; King; Koide; Kuhbock; Lavoura; Lin; Ma; Machado; Malinsky; Matsuzaki; de Medeiros Varzielas; Meloni; LM; Mitra; Molinaro; Morisi; Nardi; Parida; Paris; Petcov; Pleitez; Picariello; Rajasekaran; Riazzudin, Romao; Serodio; Skadhauge; Tanimoto; Torrente-Lujan; Urbano; Valle; Villanova del Moral; Volkas; Yin; Zee; ...;

S_4, T', $\Delta(3n^2)$: de Adelhart Toorop; Altarelli, Bazzocchi; Chen; Ding; Hagedorn; Feruglio; Frampton; Kephart; King; Lam; Lin; Luhn; Ma; Mahanthappa; Matsuzaki; de Medeiros Varzielas; LM; Morisi; Nasri; Ramond; Ross;

...]

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]



A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant (Subgroup of SO(3)).
It has 12 elements and 4 representations: 3, 1, 1', 1''

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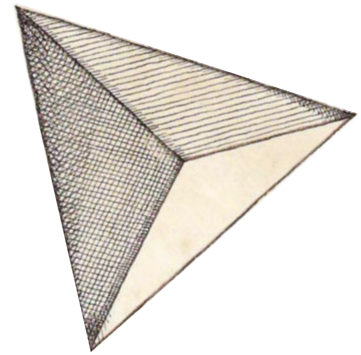

$$G_f = A_4 \times G_{aux}$$

- keeps separated the two sectors
- explains the hierarchy

$$m_e \ll m_\mu \ll m_\tau$$

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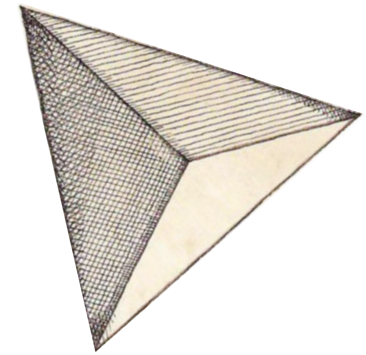

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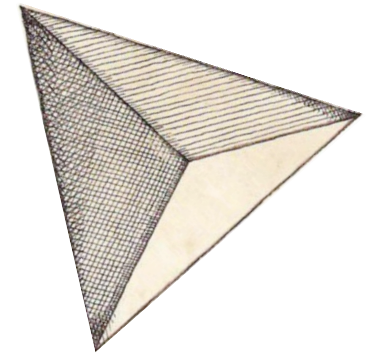

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$\langle \varphi_\ell \rangle$ → $G_\ell = Z_3$ → M_e^{diag}

$\langle \varphi_\nu \rangle$ → $G_\nu = Z_2$ → M_ν^{TB}

$$\Rightarrow \quad U = U_e^\dagger U_\nu = U^{TB}$$

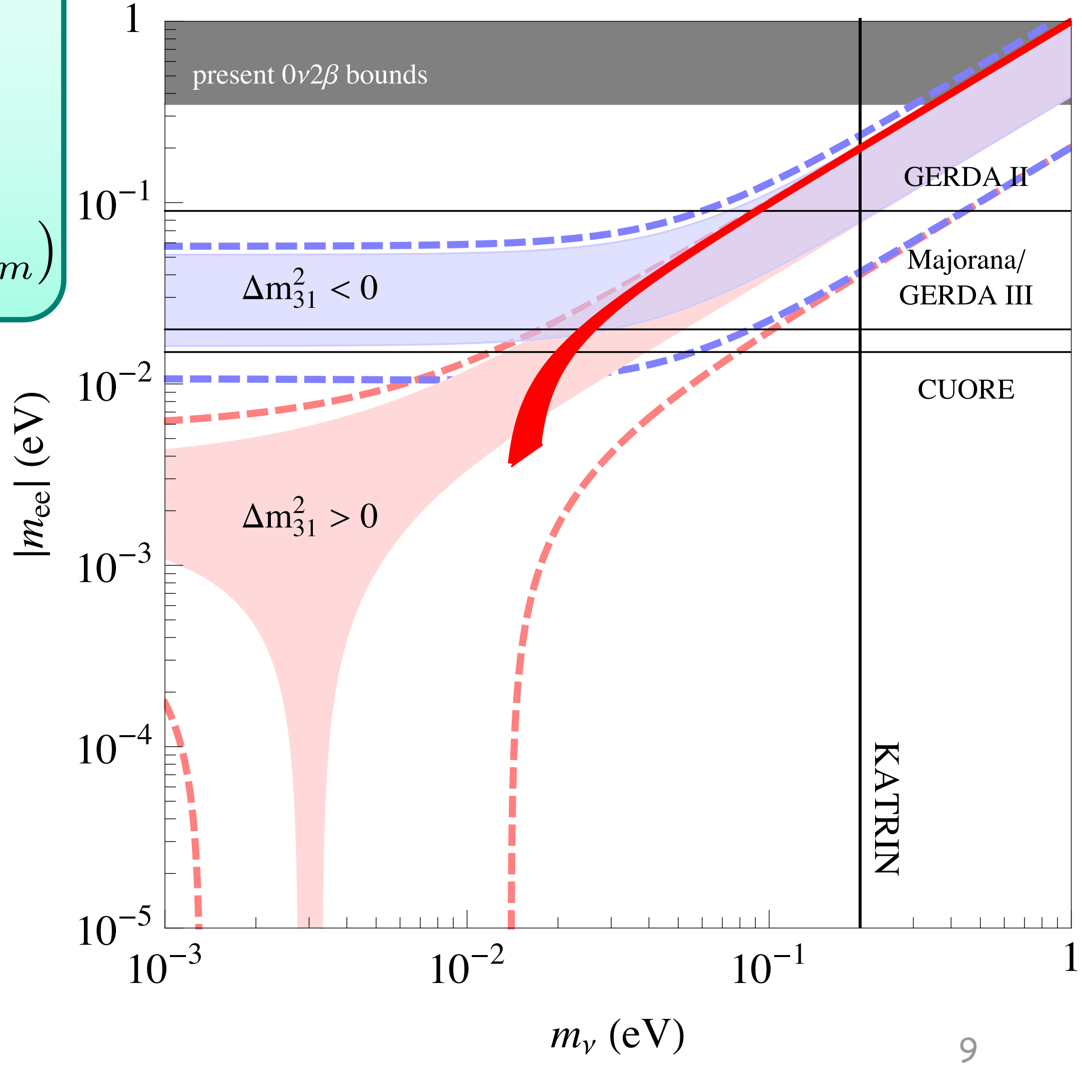
LO neutrino spectrum:

Only Normal Hierarchy

$$m_1 \geq 0.014\,\mathrm{eV}$$

$$\sum m_i \geq 0.09\,\mathrm{eV}$$

$$|m_{ee}|^2 = \frac{1}{9}\left(9m_1^2 + 5\Delta m^2_{sol} - \Delta m^2_{atm}\right)$$



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In a typical model, the corrections are democratic in all the angles:

$$\delta_{12} \approx \delta_{23} \approx \delta_{13}$$

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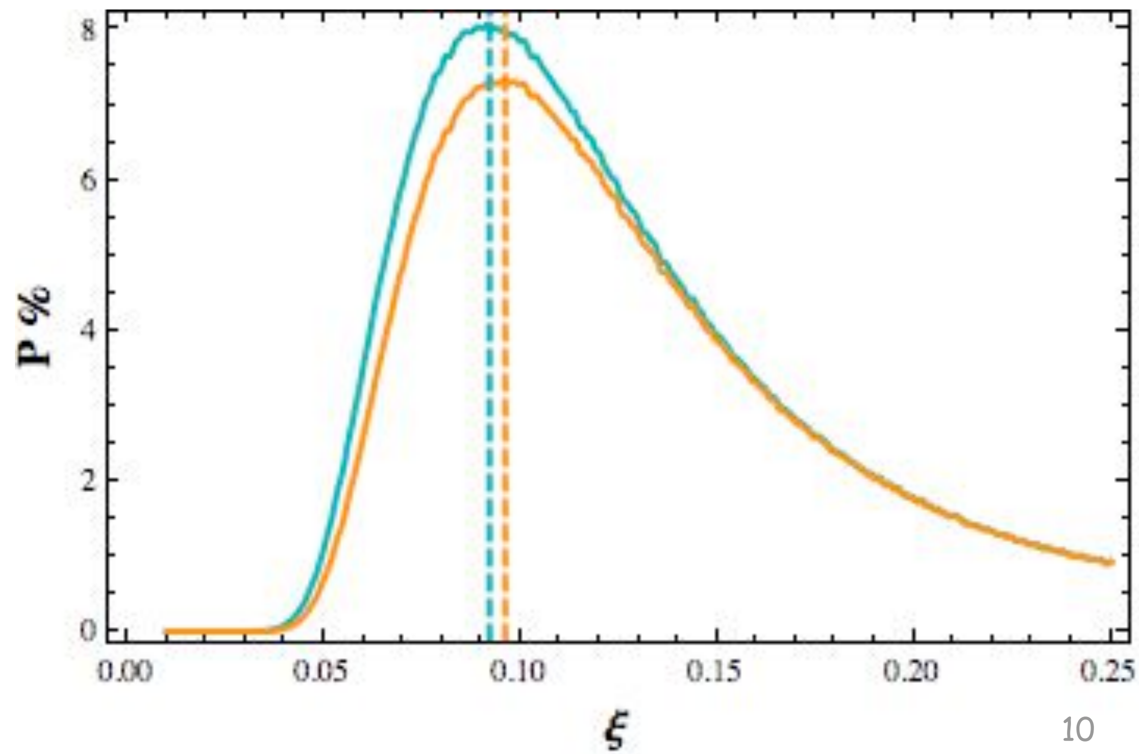
To maximize the success rate for all the three mixing angles inside the 3σ:

$$\delta_{ij} \approx 0.09$$

[Altarelli, Feruglio, LM & Stamou, to appear]



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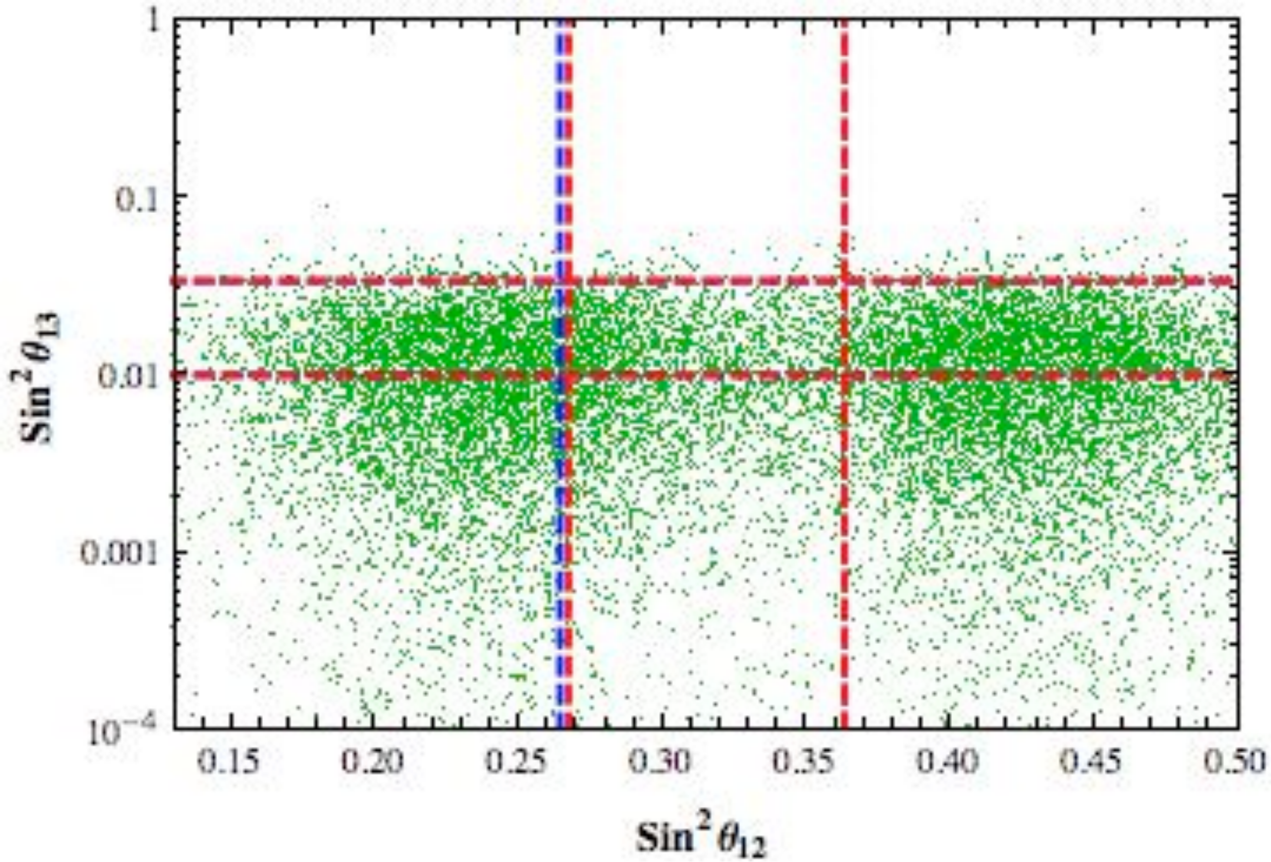

However, not so good for the **solar angle**!!!

Similar conclusions are valid for the Golden Ratio pattern.

[Feruglio and Paris 2011]

Special Tri-Bimaximal

[Lin 2009]

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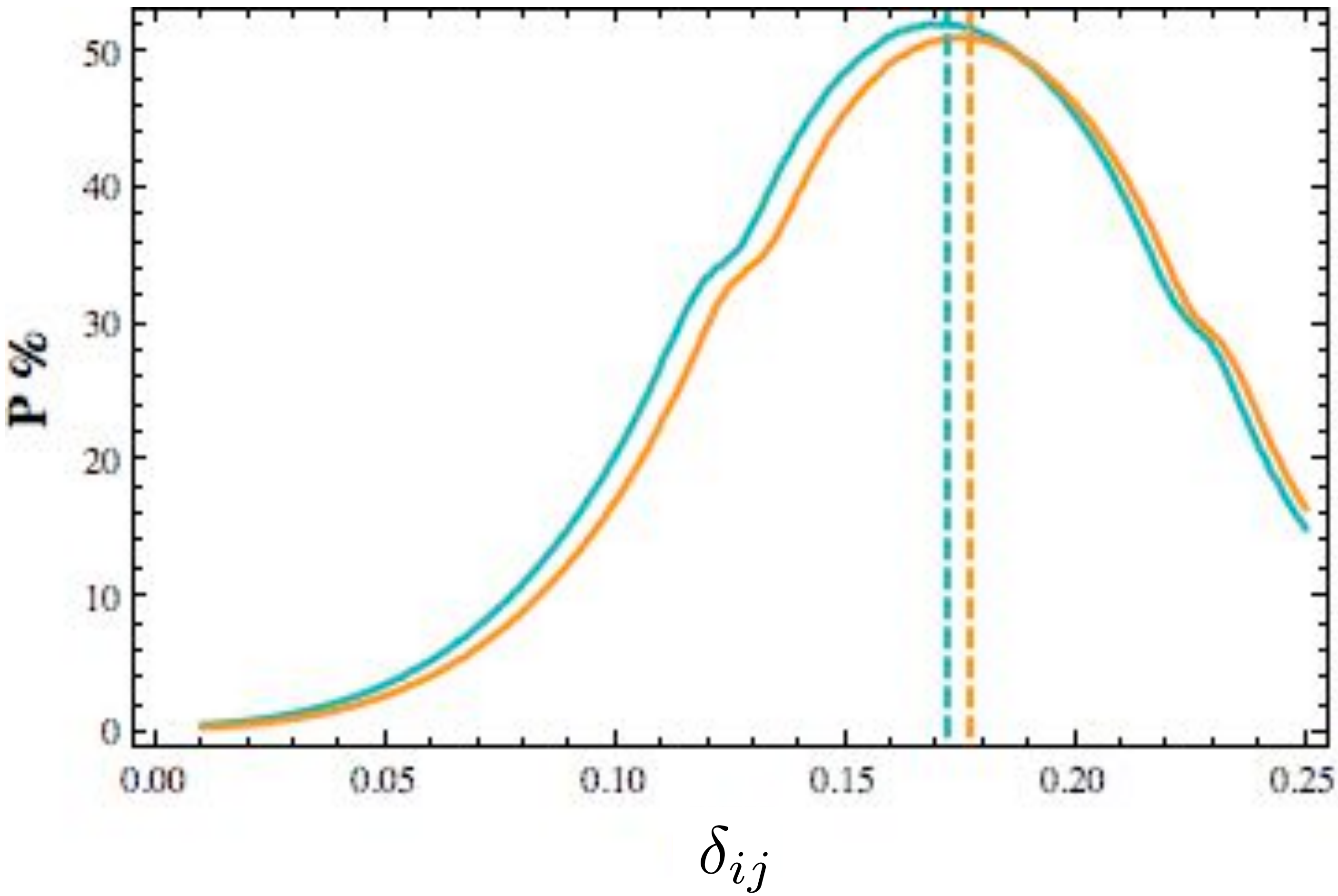
@ NNLO:

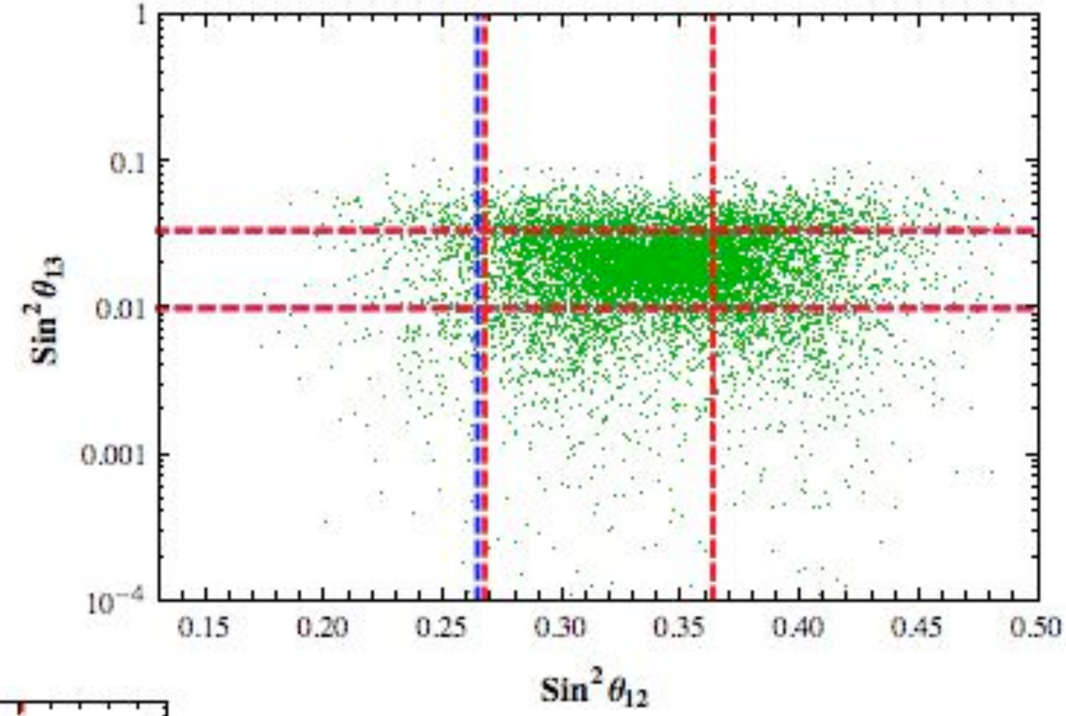
Generic corrections to all the angles $\longrightarrow$

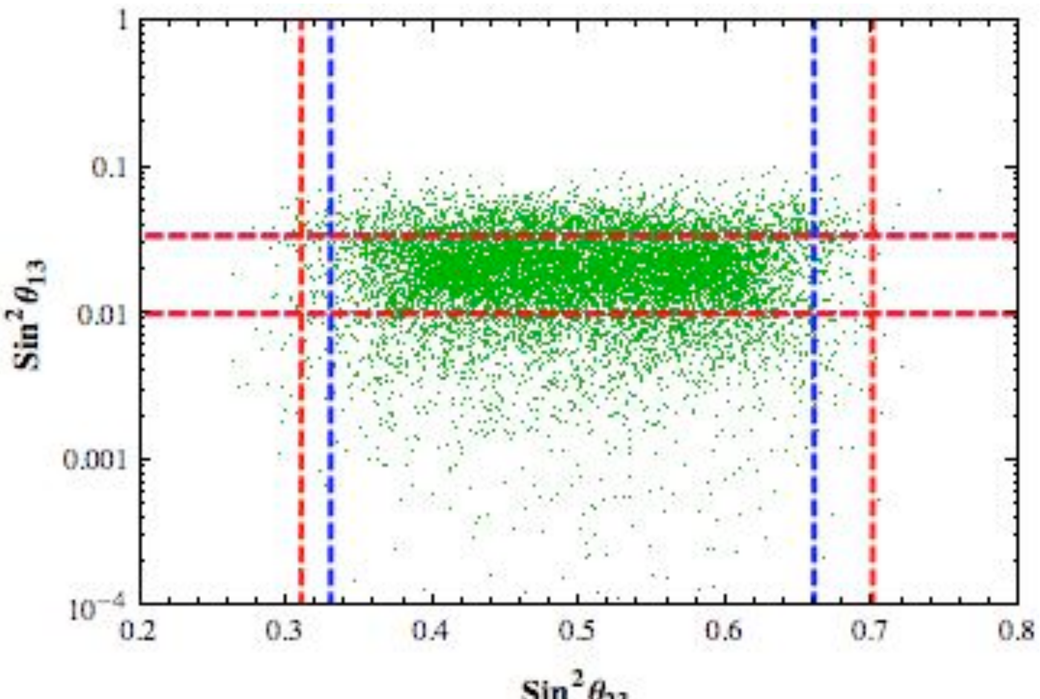
$$\theta_{13} \approx \mathcal{O}(\delta)$$
$$\sin^2\theta_{12} - \sin^2\theta_{12}^{TB} \approx \mathcal{O}(\delta^2)$$

To maximize the success rate for all the three mixing angles inside the 3σ:

$$\delta_{ij} \approx 0.172$$




1
0.1
0.01
0.001
10^{-4}
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
$\sin^2\theta_{13}$
$\sin^2\theta_{12}$


1
0.1
0.01
0.001
10^{-4}
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\sin^2\theta_{13}$
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Bimaximal

@ LO:

[Altarelli, Feruglio and LM 2009]

$$\begin{aligned} \sin^2\theta_{12}^{BM} &= 1/2 \\ \sin^2\theta_{23}^{BM} &= 1/2 \\ \sin\theta_{13}^{BM} &= 0 \end{aligned}$$

$\longrightarrow$

$$U^{BM} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & +1/\sqrt{2} \end{pmatrix}$$

$$M_\ell = M_\ell^{\text{diag}} \qquad M_\nu^{BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

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@ NLO:

$$M_\nu = M_\nu^{BM}$$

$$M_\ell = \begin{pmatrix} m_e & \epsilon\, m_e & \epsilon\, m_e \\ \epsilon\, m_\mu & m_\mu & 0 \\ \epsilon\, m_\tau & 0 & m_\tau \end{pmatrix} \longrightarrow$$

$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}(\delta_{12}+\delta_{13})$$

$$\sin^2\theta_{23} = \frac{1}{2}$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}(\delta_{12}-\delta_{13})$$

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@ NNLO:

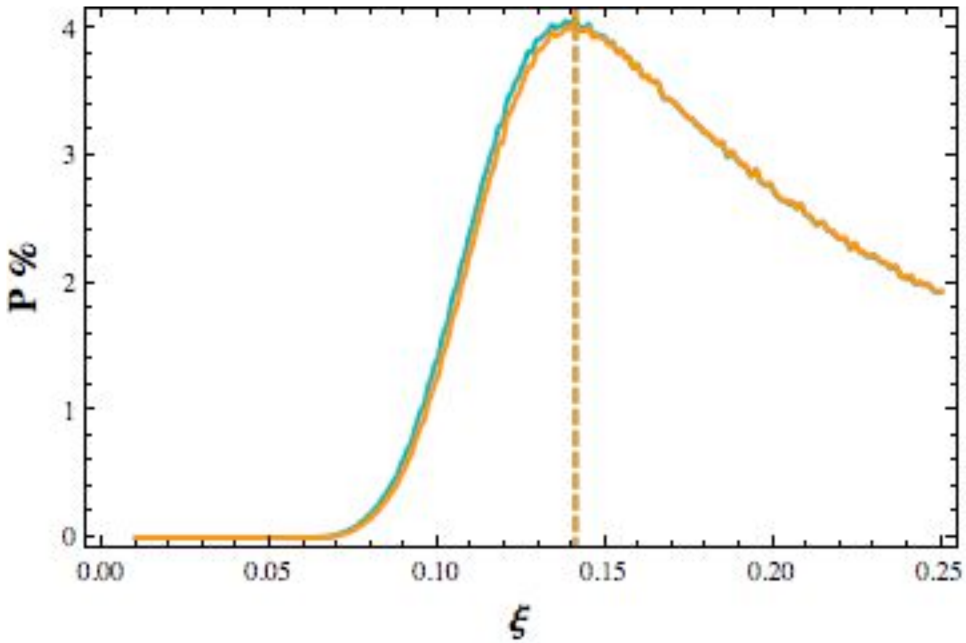
Generic corrections to all the angles $\longrightarrow$

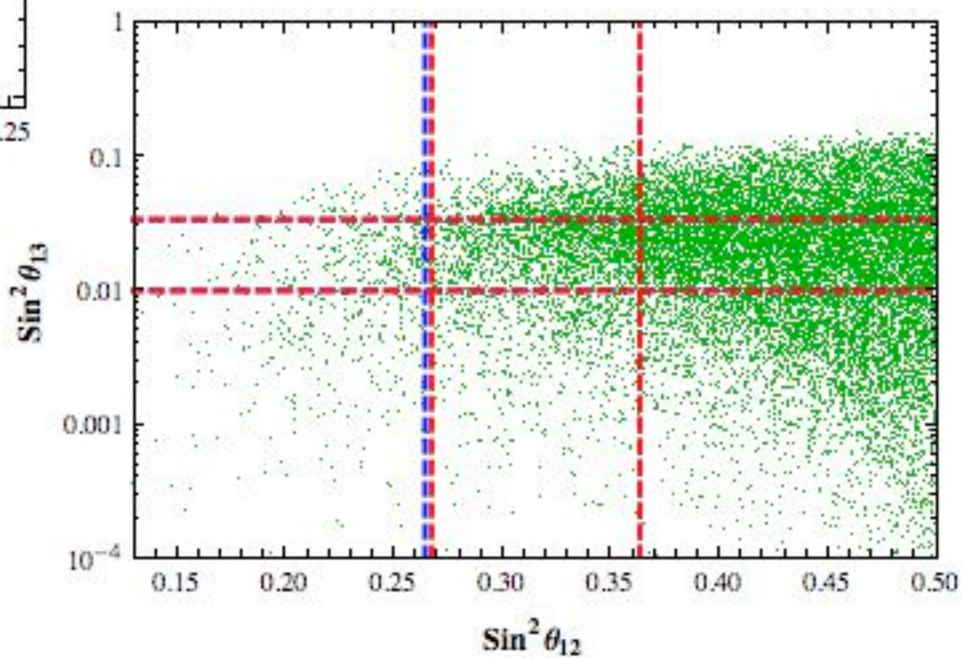
$$\theta_{13} \approx \mathcal{O}(\delta)$$

$$\sin^2\theta_{12} = 1/2 - \mathcal{O}(\delta)$$

The model is not in agreement with the data and large corrections are necessary.

The question is weather the model can arrange both the **solar** and the **reactor angles** inside their own 90% C.L. at the same time: best result with $\delta_{ij} \approx 0.141$





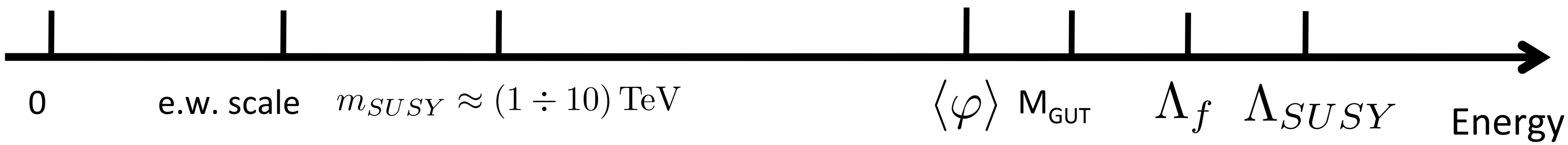
Impact on LFV

Low Energy Observables:

- ν masses
- ν oscillations

- $(g-2)_\mu$ discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

- GUTs
- flavour symmetries
- ν^c
- superheavy gauge bosons



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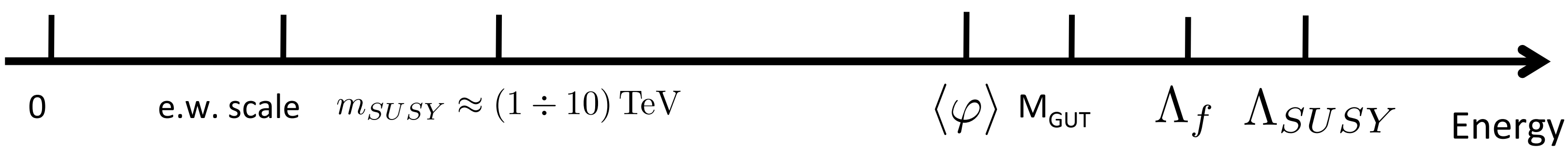

The Flavour symmetry at the High-scale affects the low-energy observables **indirectly**:

- the flavons φ do not lead to direct contributions (tree-level or loops are suppressed by the heavy mass)
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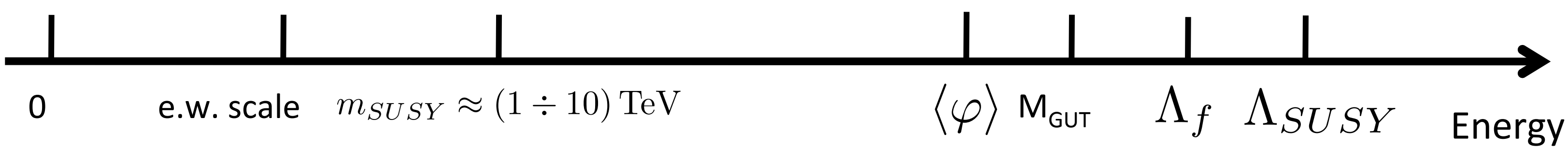

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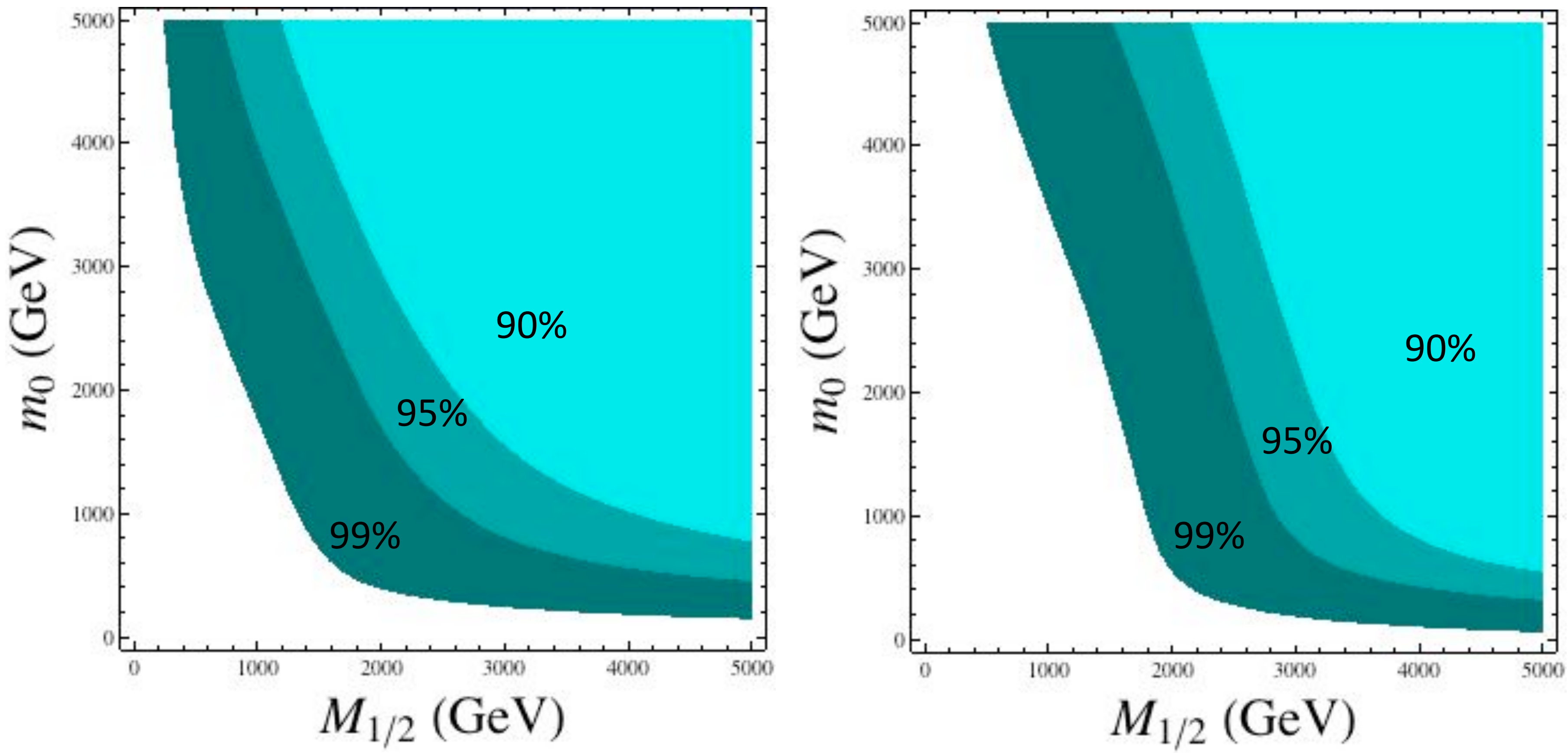
→ non-universal boundary conditions for the soft terms

→ different results wrt m-SUGRA scenario

Typical Tri-Bimaximal

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$ @ 90% C.L. [MEG coll. 2011]

$\tan\beta = 2$ $\tan\beta = 15$



Special Tri-Bimaximal

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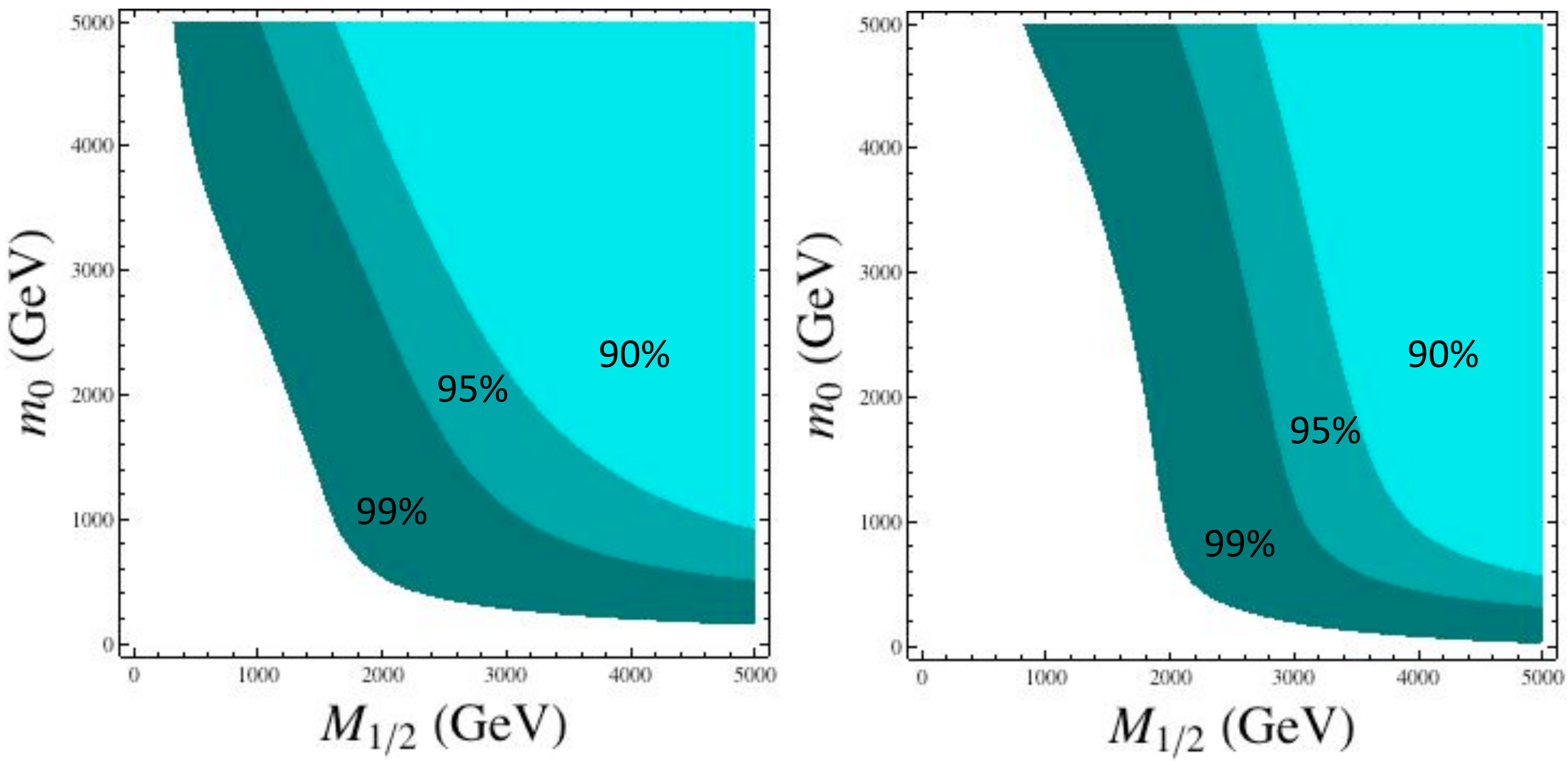
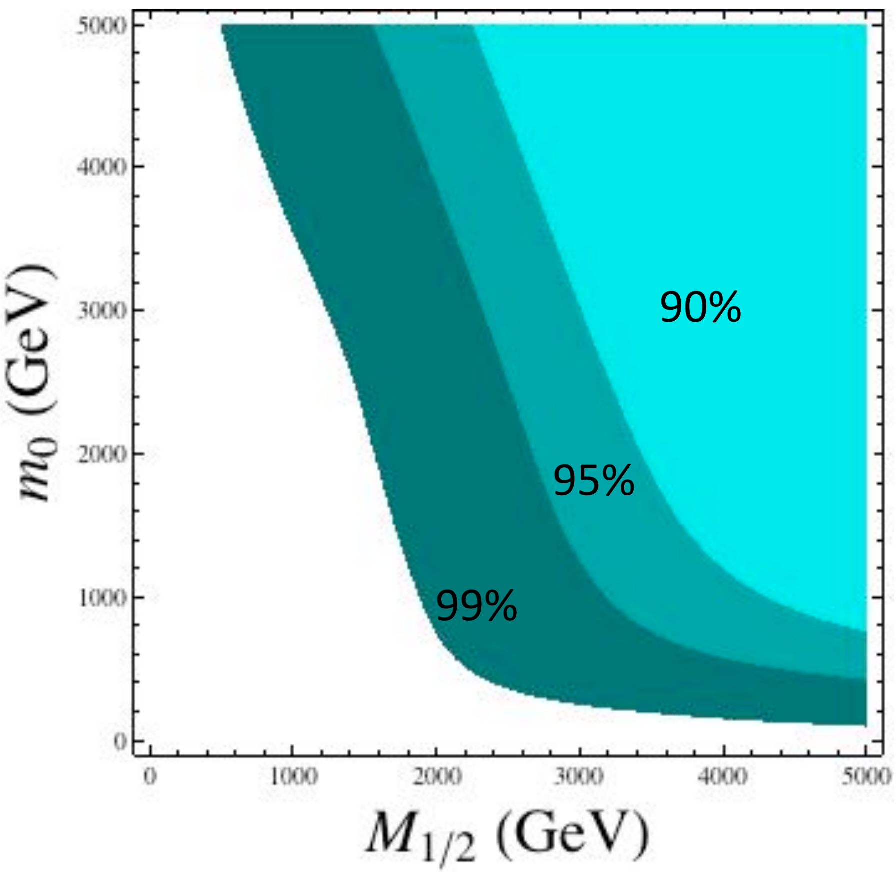
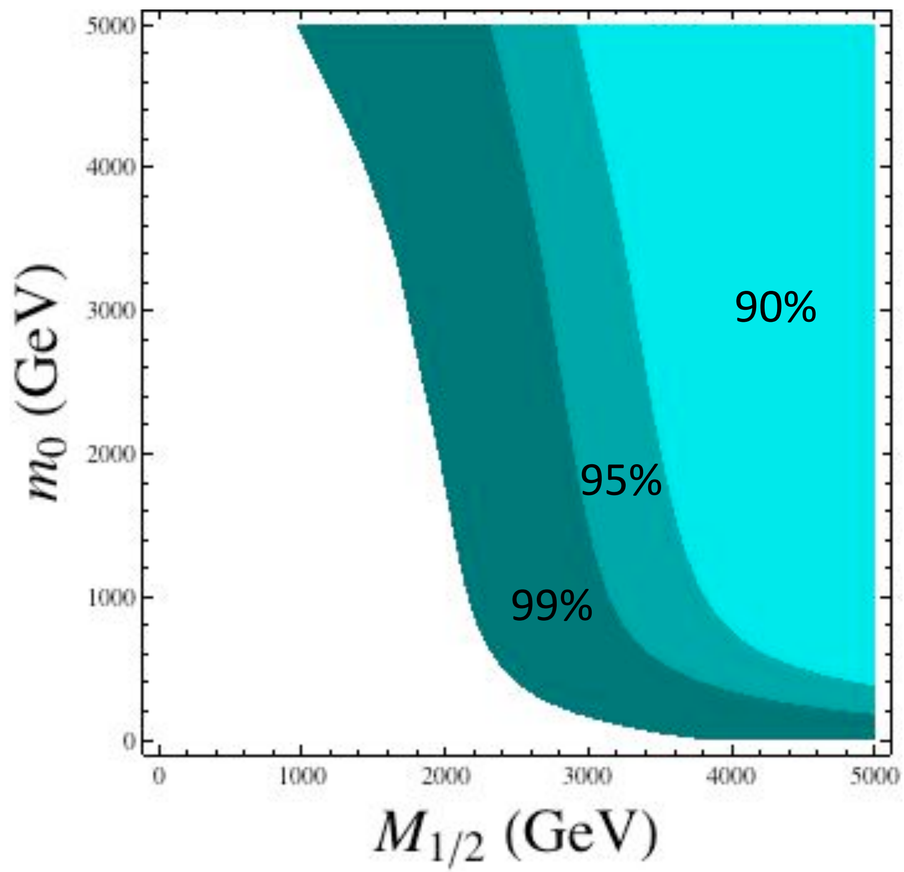


Bimaximal

$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$ @ 90% C.L. [MEG coll. 2011]

$\tan\beta = 3$ $\tan\beta = 23$





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→ **Still a lot of work to find the final neutrino flavour model**

Thanks for your attention

[Fogli *et al.* 2011]

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→ Different definition of the atmospheric mass square difference:

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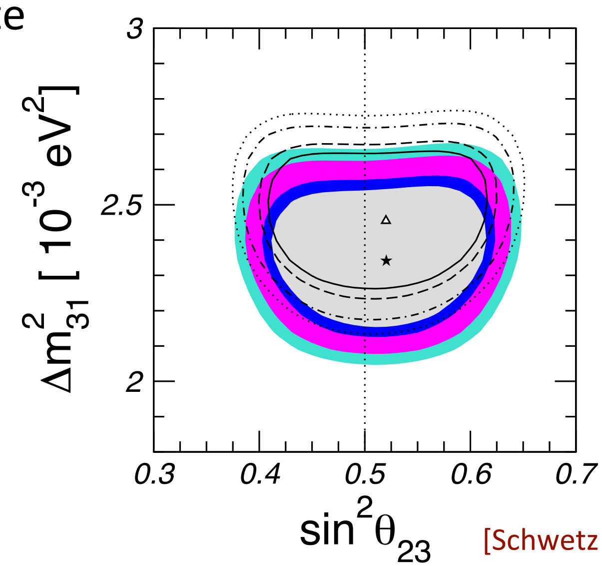
→ The maximal angle solution is still allowed and the two minima are almost degenerate



in black the NH
in colours the IH

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- different numerical results for $\sin^2\theta_{13}$

→ After the re-evaluation of the reactor neutrino fluxes, **tension** with SBL experiments (<100 m): maybe sterile neutrino with $\Delta m^2 \approx \mathcal{O}(1)\,{\rm eV}^2$ [Giunti & Laveder 2011]

→ Fogli *et al.* do not consider these SBL data set, while Schwetz *et al.* do

→ Schwetz *et al.* give also the results when SBL data are not considered: much more similar to Fogli *et al.*

$$\sin^2\theta_{13} = 0.022 \pm 0.008\,[0.026 \pm 0.009]$$

Numerical Accidents?

Are these patterns only numerical accidents? If **Yes** what?

→ **Anarchy (?)** [Review: Altarelli, Feruglio & Masina 2002] [Recentrly: Buchmuller, Domcke & Schmitz 2011]

Consist in using a simple U(1) as flavour symmetry:

- many parameters already at the LO
- low predictive power
- no correlations among observables

→ but the mixing angles and the r parameter can be accommodated

$$SU(5)\times U(1)$$

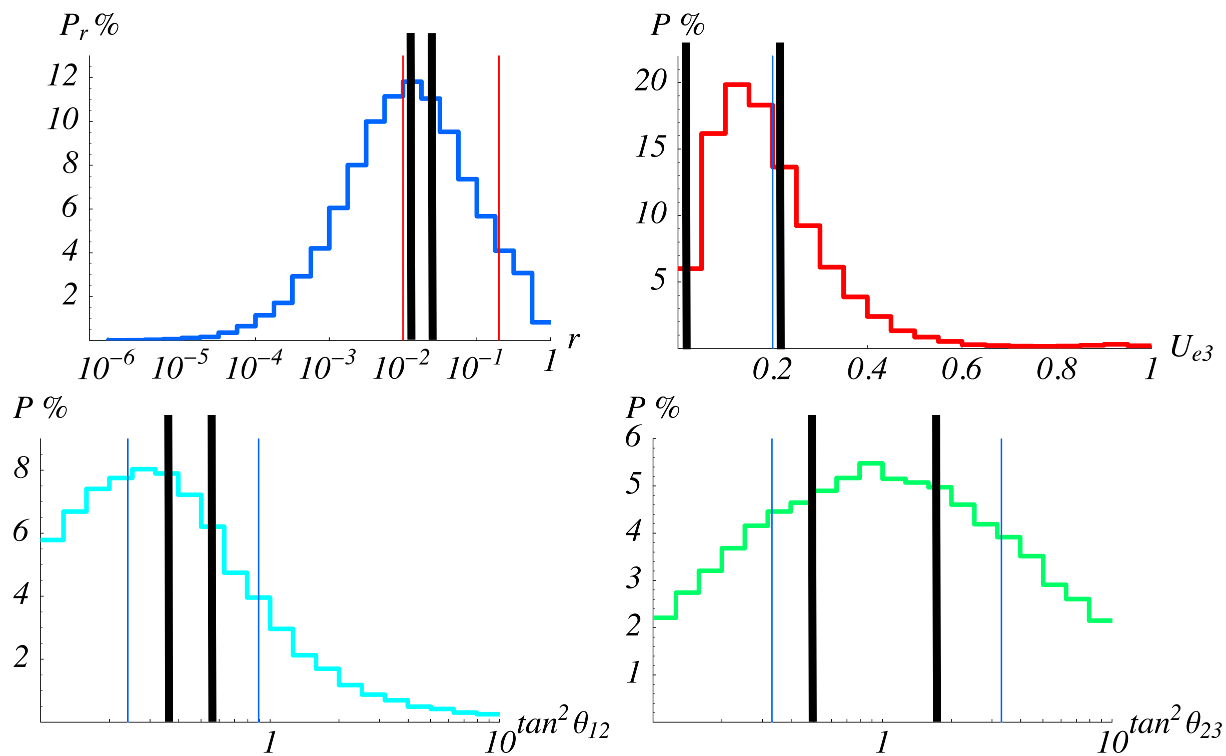
$$\Psi_{10}=(5,3,0)$$

$$\Psi_{\overline{5}}=(2,0,0)$$

$$\Psi_1=(1,-1,0)$$

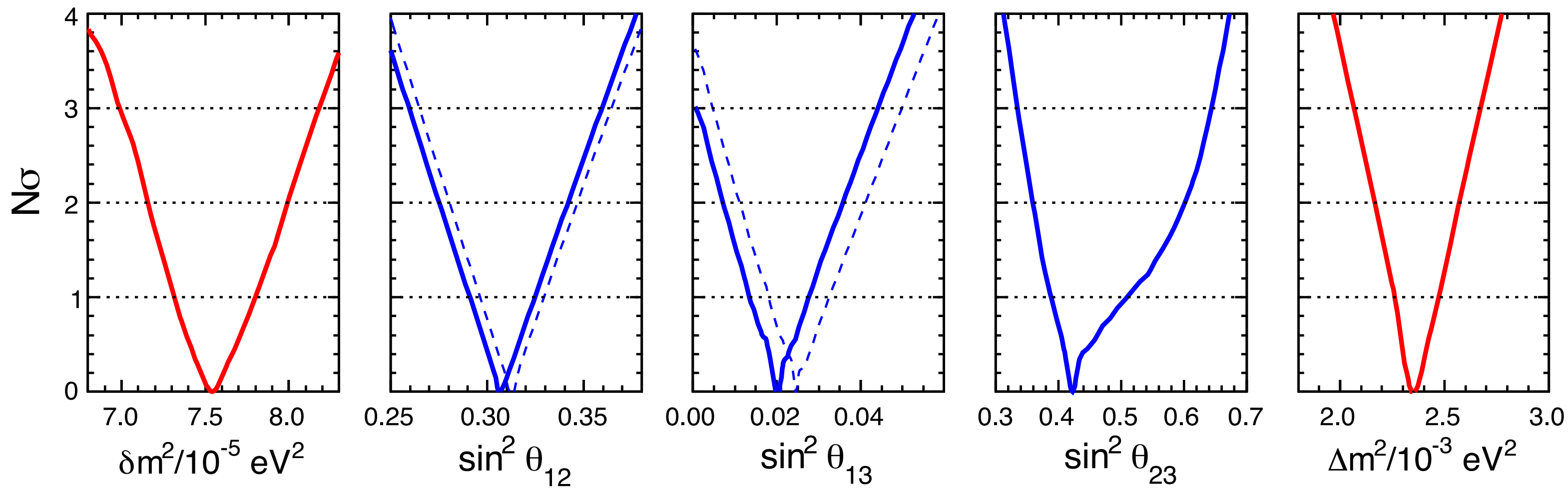
$$m_\nu=\begin{pmatrix}\epsilon^4 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & 0 & 0\\ \epsilon^2 & 0 & 0\end{pmatrix}$$

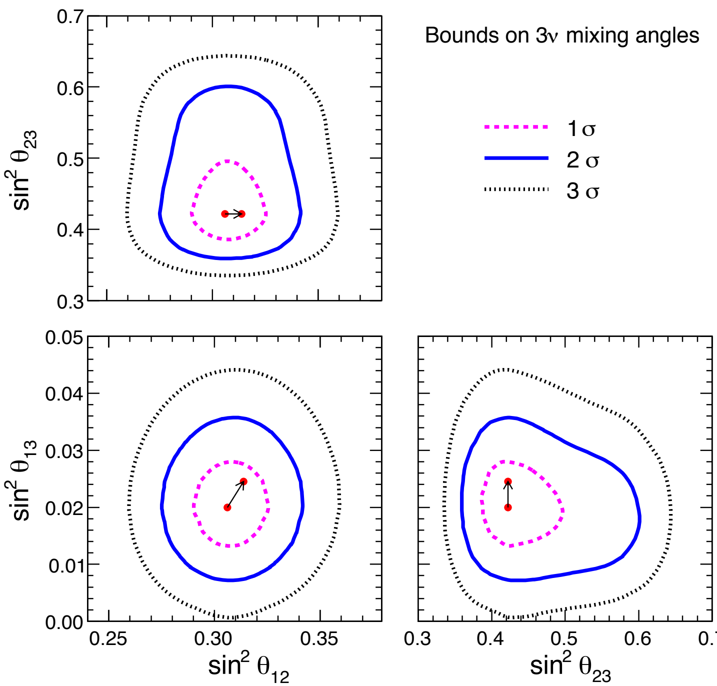
$$\epsilon\approx 0.5$$

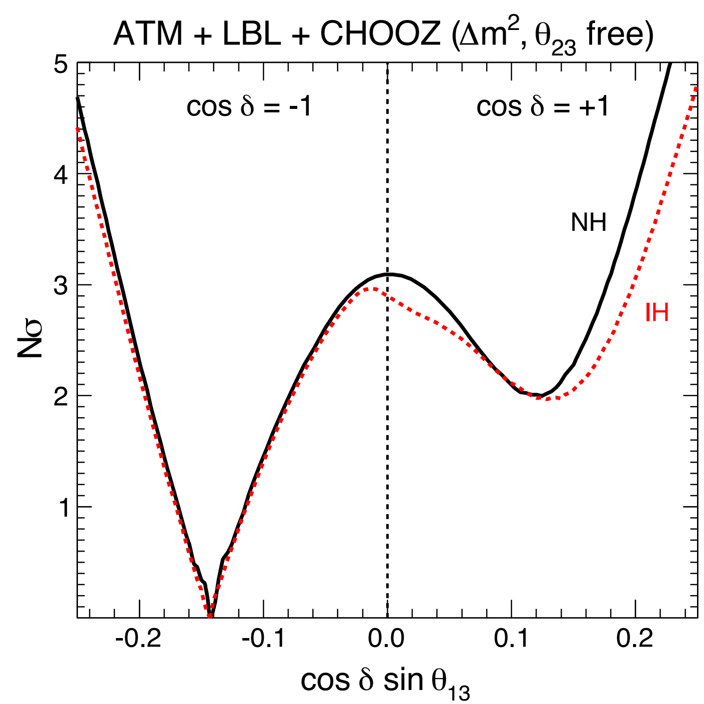


Neutrino Oscillation Fits

[Fogli *et al.* 2011]







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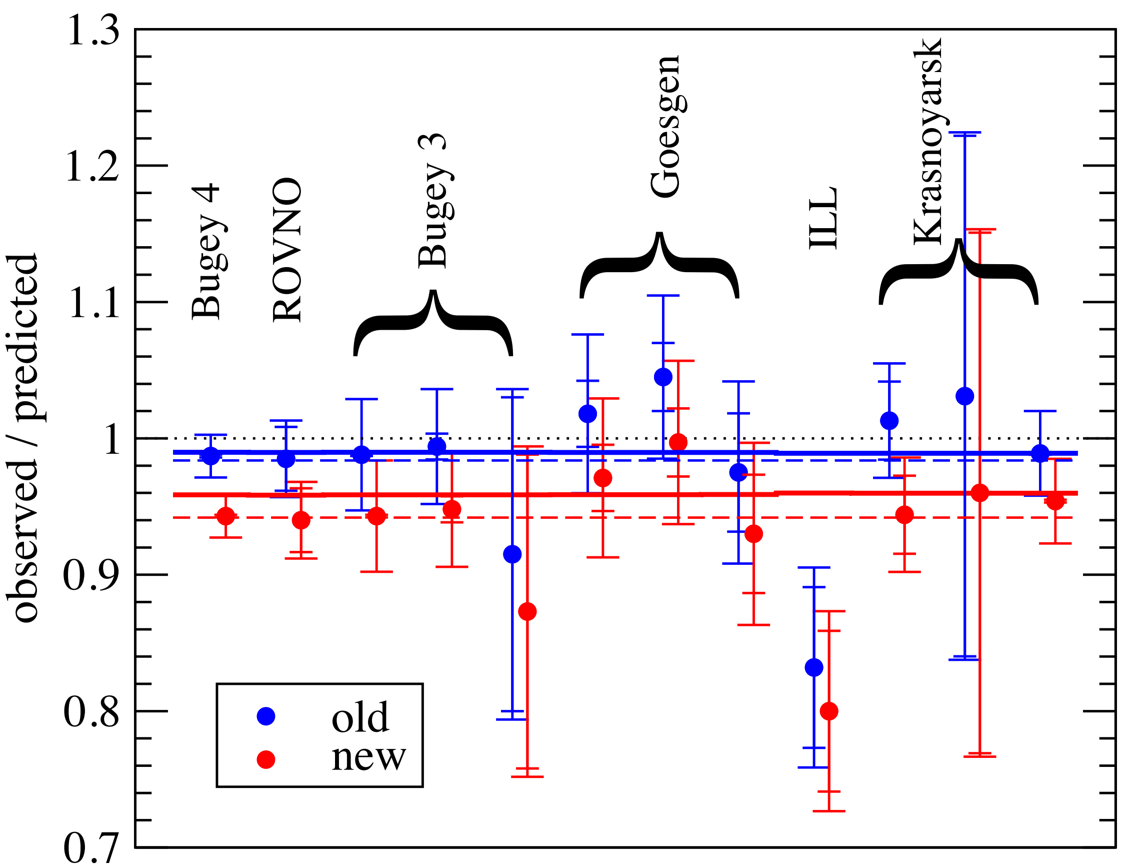
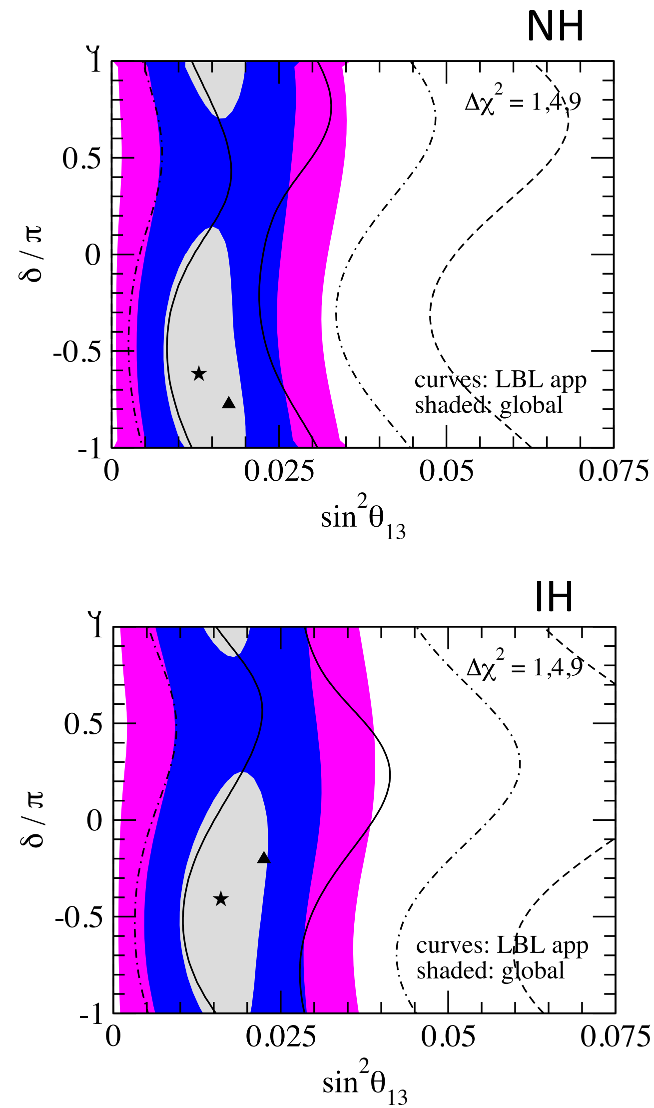




The Flavour Puzzle

Charged Fermions

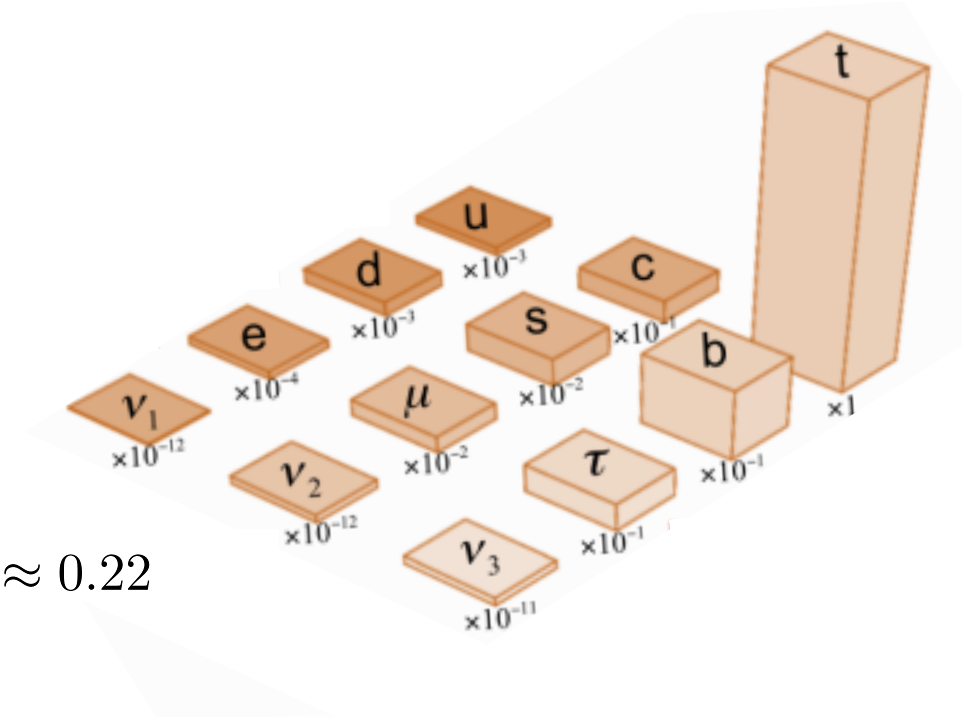
$$m_u : m_c : m_t \approx \lambda^{7\div8} : \lambda^3 : 1$$

$$m_d : m_s : m_b \approx \lambda^{4\div5} : \lambda^{2.5} : 1$$

$$m_e : m_\mu : m_\tau \approx \lambda^5 : \lambda^2 : 1$$

$$m_\tau : m_b : m_t \approx \lambda^3 : \lambda^{2.5} : 1$$

$$\lambda \approx \theta_C \approx 0.22$$



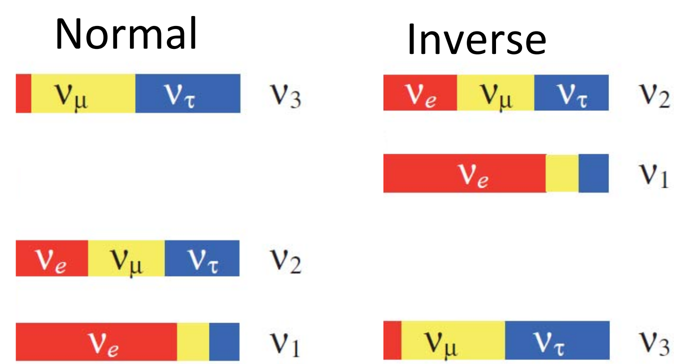
Neutrinos

Considering the conservative case with 3 active neutrinos:

$$m_\nu \lesssim \mathcal{O}(\text{eV})$$

$$\Delta m^2_{sol} \equiv m_2^2 - m_1^2 \sim 2 \times 10^{-5}\,\text{eV}^2$$

$$\left|\Delta m^2_{atm}\right| \equiv \left|m_3^2 - m_1^2\right| \sim 8 \times 10^{-3}\,\text{eV}^2$$



The Mixing Matrices

CKM Matrix

$$V = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}, \delta) \cdot R_{12}(\theta_{12})$$

$$\begin{aligned} \sin\theta_{12} &\simeq 0.22 \\ \sin\theta_{23} &\simeq 0.04 \\ \sin\theta_{13} &\simeq 0.004 \\ \delta &\simeq 77^\circ \end{aligned}$$

PMNS Matrix (end of 2008)

Atmospheric → $R_{23}(\theta_{23})$; Reactor → R_{13}; Dirac → δ; Solar → $R_{12}(\theta_{12})$; Majorana → P

$$U = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}, \delta) \cdot R_{12}(\theta_{12}) \cdot P$$

[Fogli *et al.* 0809.2936]

$$\begin{aligned} \sin^2\theta_{12} &= 0.312^{+0.019}_{-0.018} \quad (0.26 - 0.37) \\ \sin^2\theta_{23} &= 0.466^{+0.073}_{-0.058} \quad (0.331 - 0.644) \\ \sin^2\theta_{13} &= 0.020^{+0.016}_{-0.016} \quad (\leq 0.046) \end{aligned}$$

$\theta_{12} \approx 34^\circ$

$\theta_{23} \approx 43^\circ$

$\theta_{13} \approx 8^\circ$

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]

	Matter fields				Higgs		Flavons		
	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	θ	φ_T	φ_S	ξ
A_4	3	1	$1''$	$1'$	1	1	3	3	1

$$w_e = y_e \frac{\theta^2}{\Lambda^3} e^c (\varphi_T \ell) h_d + y_\mu \frac{\theta}{\Lambda^2} \mu^c (\varphi_T \ell)' h_d + y_\tau \frac{1}{\Lambda} \tau^c (\varphi_T \ell)'' h_d$$

$$w_\nu = x_a \frac{\xi}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} + x_b \left(\frac{\varphi_S}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} \right)$$

Expansion in ϕ/Λ

vacuum alignment:

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (u, 0, 0)$$

$$\frac{\langle \varphi_S \rangle}{\Lambda} = c_b (u, u, u)$$

$$\frac{\langle \xi \rangle}{\Lambda} = c_a u$$

$$\frac{\langle \theta \rangle}{\Lambda} = t$$

$$M_e = \mathrm{diag}(y_e\, t^2,\, y_\mu\, t,\, y_\tau)\, v_d\, u$$

$$\frac{m_e}{m_\mu} = \frac{m_\mu}{m_\tau} = t \approx 0.05$$

$$M_\nu = \begin{pmatrix} a + \frac{2}{3} b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3} b & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2}{3} b \end{pmatrix} v_u^2$$

$$M_\nu^{diag} = v_u^2 \mathrm{diag}(a + b,\, a,\, -a + b)$$

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$$

Radiative Lepton Decays

$$-\mathcal{L}_m \supset \quad \begin{pmatrix} \bar{\tilde{e}} & \tilde{e}^c \end{pmatrix} \mathcal{M}_e^2 \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}}^c \end{pmatrix} + \bar{\tilde{\nu}}\, m^2_{\nu LL}\, \tilde{\nu}$$

$$\mathcal{M}_e^2 = \begin{pmatrix} m^2_{eLL} & m^2_{eLR} \\ m^2_{eRL} & m^2_{eRR} \end{pmatrix}$$

$$R_{ij} = \frac{48\pi^3\alpha}{G_F^2 m^4_{SUSY}} \left(|A_L^{ij}|^2 + |A_R^{ij}|^2 \right)$$

$$\begin{aligned} A_L^{ij} &= \underline{a_{LL}(\delta_{ij})_{LL}} + a_{RL}\frac{m_{SUSY}}{m_i}(\delta_{ij})_{RL} \\ A_R^{ij} &= a_{RR}(\delta_{ij})_{RR} + a_{LR}\frac{m_{SUSY}}{m_i}(\delta_{ij})_{LR} \end{aligned}$$

$$\tan\beta = \{2, 25\} \qquad \begin{cases} a_{LL} = \{2, 27\} \\ a_{RR} = \{-1.9, -0.6\} \\ a_{RL} = a_{LR} = 0.3 \end{cases} \qquad (\delta_{ij})_{CC'} = \frac{(\hat{m}^2_{eCC'})_{ij}}{m^2_{SUSY}}$$